TRANSPARENCY MASTERS

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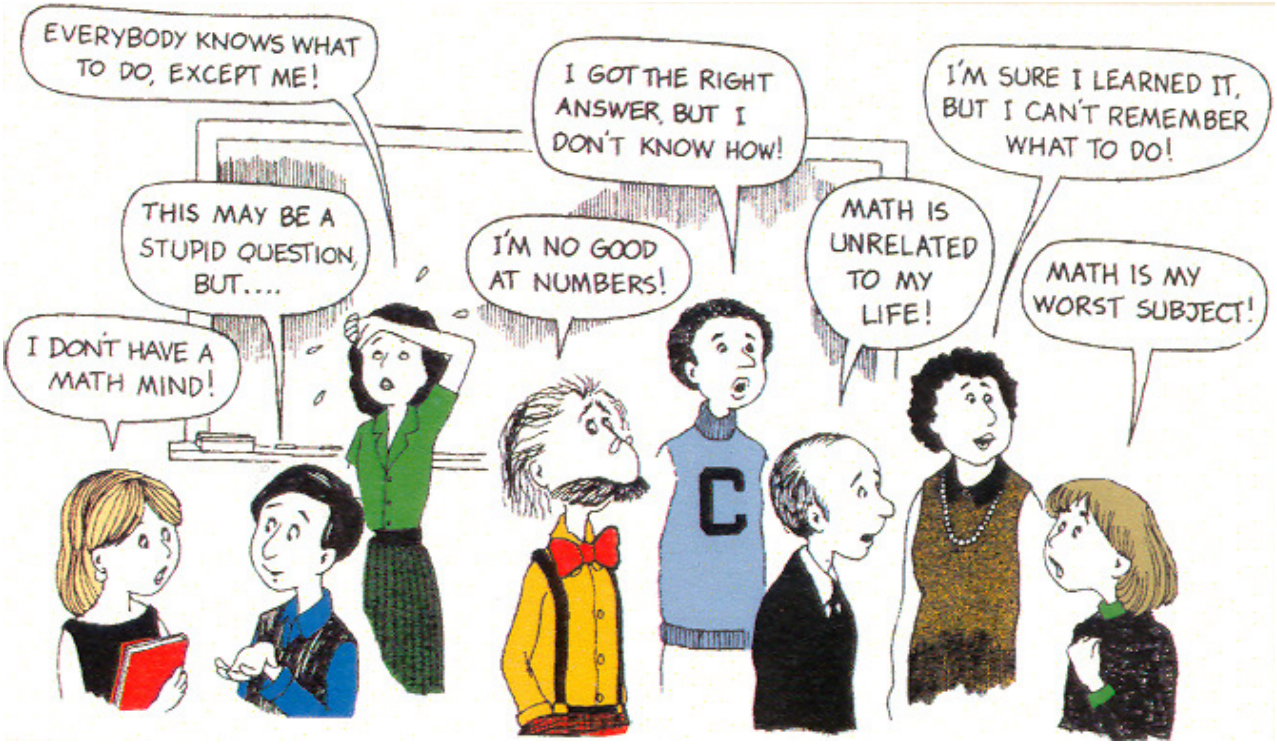
THE NATURE OF MATHEMATICS

MATH ANXIETY



Math Anxiety Bill of Rights*

by Sandra L. Davis

1. I have the right to learn at my own pace and not feel put down or stupid if I'm slower than someone else.
2. I have the right to ask whatever questions I have.
3. I have the right to need extra help.
4. I have the right to ask a teacher or TA for help.
5. I have the right to say I don't understand.
6. I have the right not to understand.
7. I have the right to feel good about myself regardless of my abilities in math.
8. I have the right not to base my self-worth on my math skills.
9. I have the right to view myself as capable of learning math.
10. I have the right to evaluate my math instructors and how they teach.
11. I have the right to relax.
12. I have the right to be treated as a competent adult.
13. I have the right to dislike math.
14. I have the right to define success in my own terms.

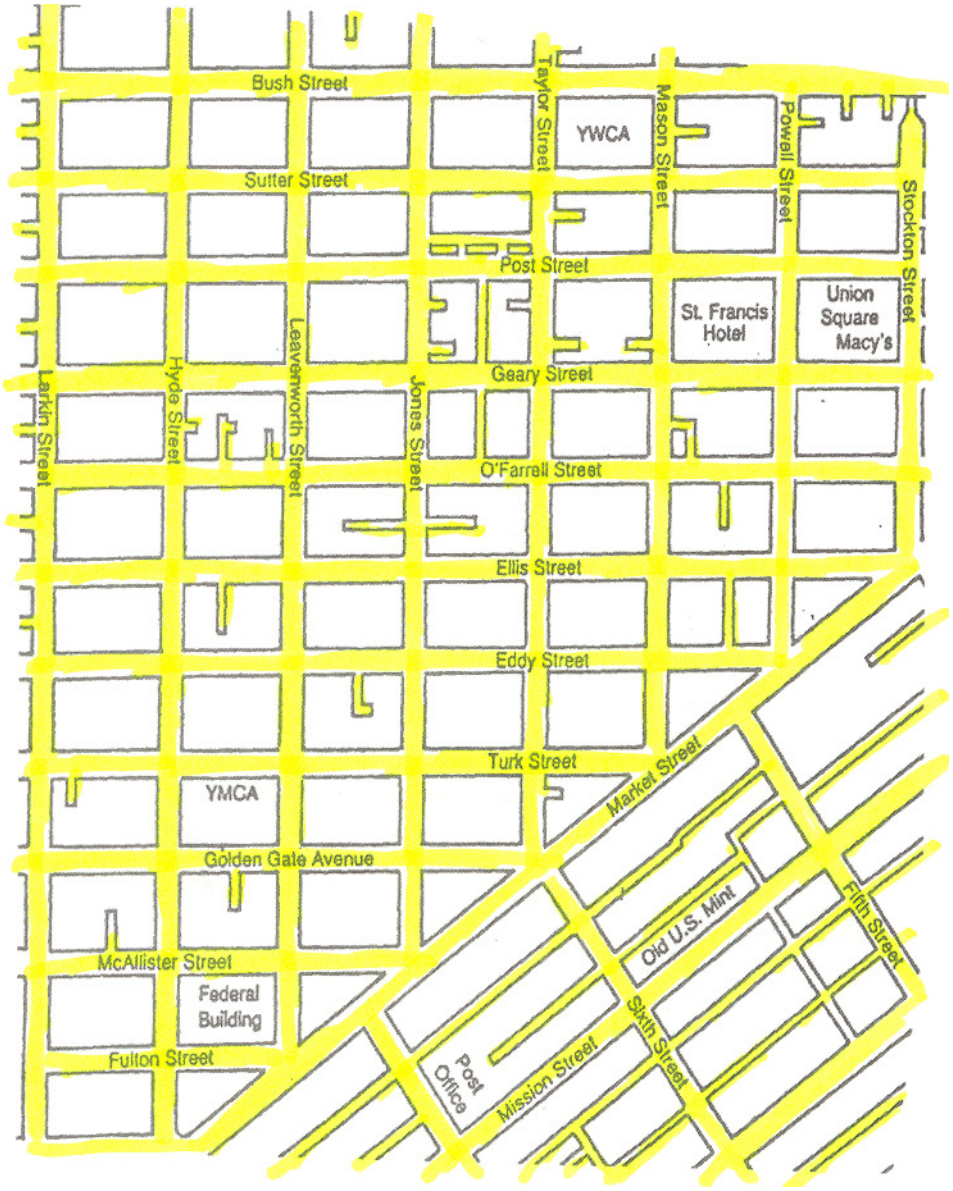
*From *Overcoming Math Anxiety*, by Sheila Tobias, pp. 236-237.

Guidelines for PROBLEM SOLVING

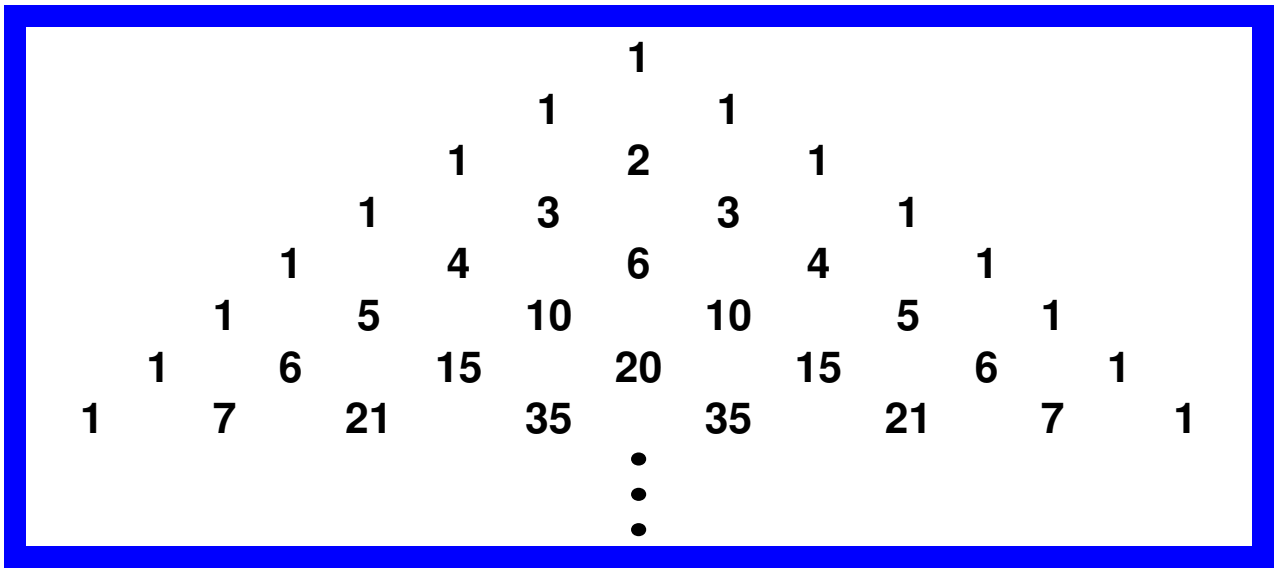
FIRST: Understand the Problem
SECOND: Devise a Plan
THIRD: Carry out the Plan
FOURTH: Look Back

1. Look for a simpler related problem
2. Work backward
3. Work forward
4. Narrow the condition
5. Widen the condition
6. Seek a counterexample
7. Guess and test
8. Divide and conquer
9. Change the conceptual mode

MAP OF SAN FRANCISCO



PASCAL'S TRIANGLE



Designate rows and columns:

$\binom{0}{0}$ ← Row Number (on top)
← Column Number (on bottom)

$\binom{1}{0}$ $\binom{1}{1}$

$\binom{2}{0}$ $\binom{2}{1}$ $\binom{2}{2}$

$\binom{3}{0}$ $\binom{3}{1}$ $\binom{3}{2}$ $\binom{3}{3}$

$\binom{4}{0}$ $\binom{4}{1}$ $\binom{4}{2}$ $\binom{4}{3}$ $\binom{4}{4}$

PATTERNS IN MULTIPLICATION

Nine

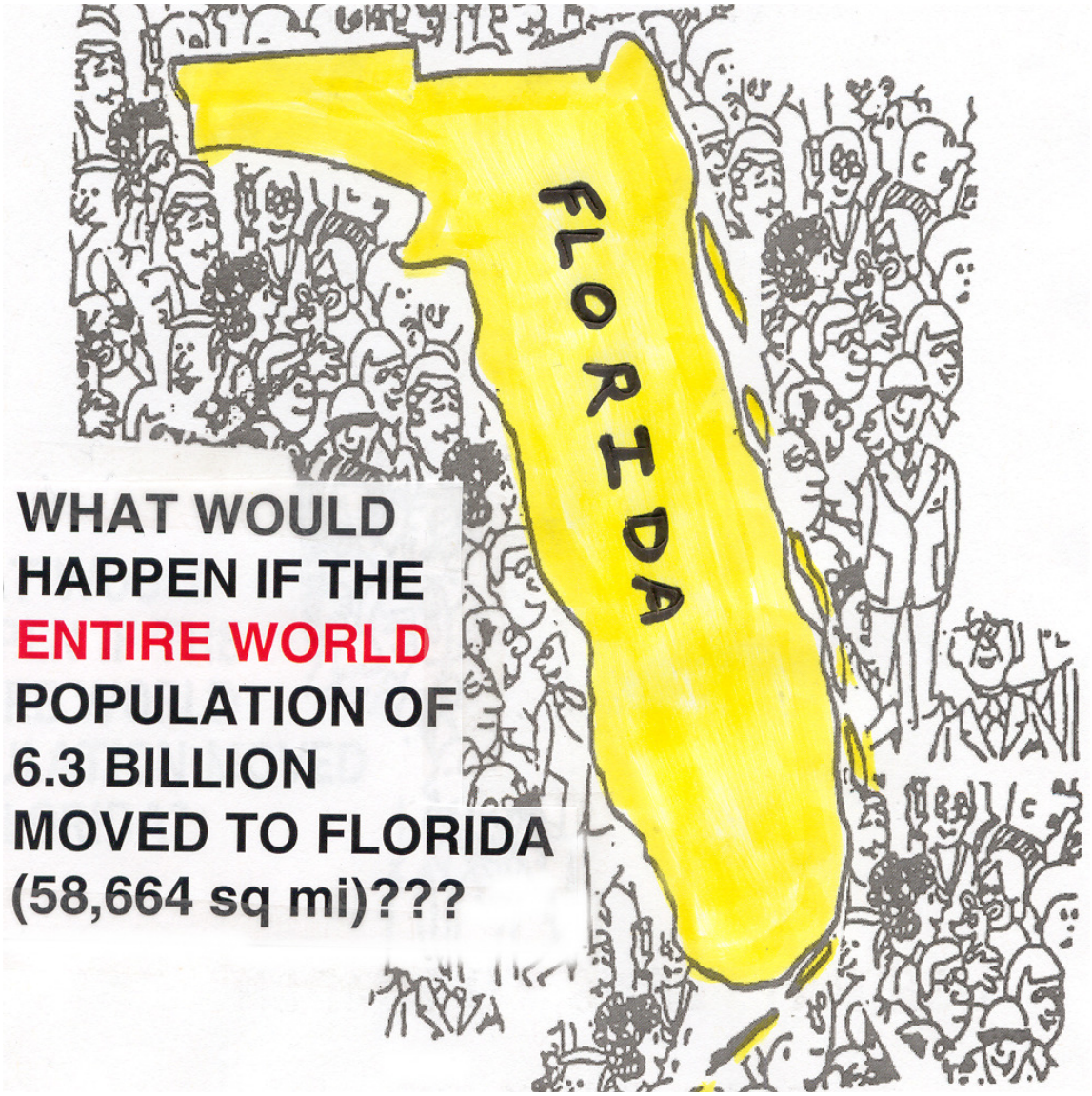
$$1 \times 9 = 9$$
$$2 \times 9 = 18$$
$$3 \times 9 = 27$$
$$4 \times 9 = 36$$
$$5 \times 9 = 45$$
$$6 \times 9 = 54$$
$$7 \times 9 = 63$$
$$8 \times 9 = 72$$
$$9 \times 9 = 81$$
$$10 \times 9 = 90$$

USE PATTERNS TO SIMPLIFY:

$$\frac{(999{,}999{,}999)(999{,}999{,}999)}{1+2+3+4+5+6+7+8+9+8+7+6+5+4+3+2+1}$$

EXTRA! EXTRA!

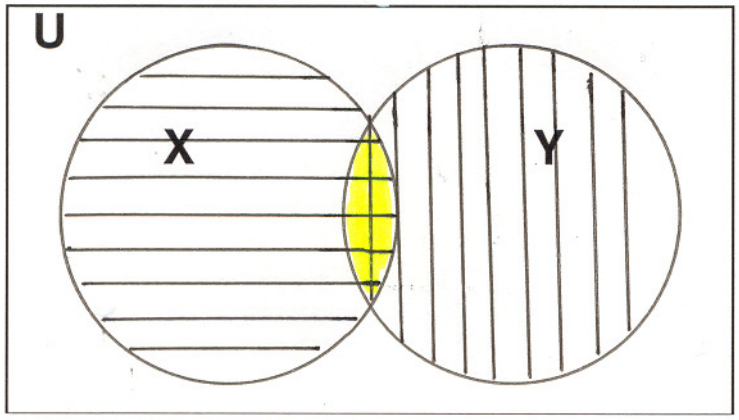
ENTIRE WORLD POPULATION MOVES TO FLORIDA



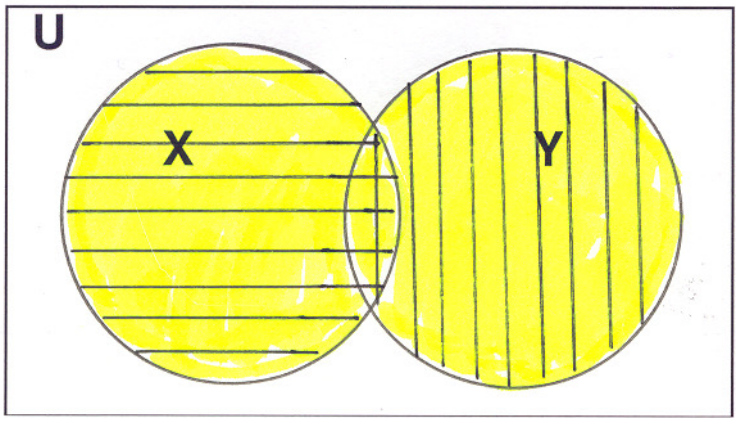
How much space would a family of four persons have?

A. 10 sq in. (oh no!)
B. 10 sq ft (standing room only)
C. 100 sq ft (a small room)
D. 1,000 sq ft (a typical apartment)
E. 10,000 sq ft (a grand estate)

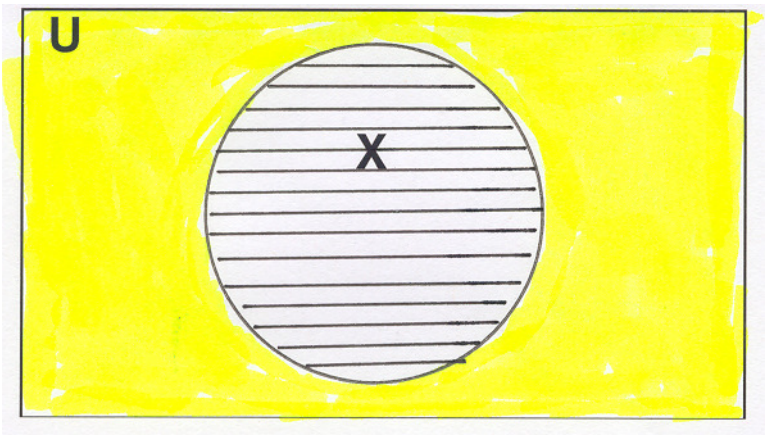
INTERSECTION, $X \cap Y$



UNION, $X \cup Y$



COMPLEMENT, $\overline{X}$



TRUTH OR CONSEQUENCES

Truth Table of the Fundamental Operators

		Conjunction	Disjunction	Negation	
p	q	$p \wedge q$	$p \vee q$	$\sim p$	$\sim q$
		AND	***OR***	***NOT***	
T	T	T	T	F	F
T	F	F	T	F	T
F	T	F	T	T	F
F	F	F	F	T	T

IF IF IF IF.....
we'd all be rich!

If *p*, then *q*.

Definition of Conditional

p	q	$p \rightarrow q$
		IMPLIES
T	T	T
T	F	F
F	T	T
F	F	T

Speak the language . . .

ADDITIONAL OPERATORS

p	q	$(p \vee q) \wedge \sim (p \wedge q)$	$\sim (p \vee q)$
		No p is q	**Neither p nor q**
T	T	F	F
T	F	T	F
F	T	T	F
F	F	F	T

p	q	$\sim q \rightarrow p$	$(p \wedge q) \wedge (q \rightarrow p)$	$p \rightarrow \sim q$
		p unless q	**p because q**	**No p is q**
T	T	T	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	F	T

NUMERATION SYSTEMS

Egyptian Hieroglyphic Numerals

Decimal Numeral	*Egyptian Numeral*	*Descriptive Name*
1		Stroke
10		Heel bone
100		Scroll
1,000		Lotus flower
10,000		Pointing finger
100,000		Polliwog
1,000,000		Astonished man

Babylonian Cuneiform Numerals

Decimal Numeral	*Babylonian Numeral*
1	
2	
9	
10	
59	

DO YOU COUNT?

How many?

a. Hindu-Arabic

b. Egyptian

c. Babylonian

d. Roman

PLACE VALUE PLEASE!

Place-value chart

Base	*Place value*					
two	2^5	2^4	2^3	2^2	2^1	$2^0 = 1$
three	3^5	3^4	3^3	3^2	3^1	$3^0 = 1$
four	4^5	4^4	4^3	4^2	4^1	$4^0 = 1$
five	5^5	5^4	5^3	5^2	5^1	$5^0 = 1$
eight	8^5	8^4	8^3	8^2	8^1	$8^0 = 1$
ten	10^5	10^4	10^3	10^2	10^1	$10^0 = 1$
twelve	12^5	12^4	12^3	12^2	12^1	$12^0 = 1$

A HUMAN COMPUTER

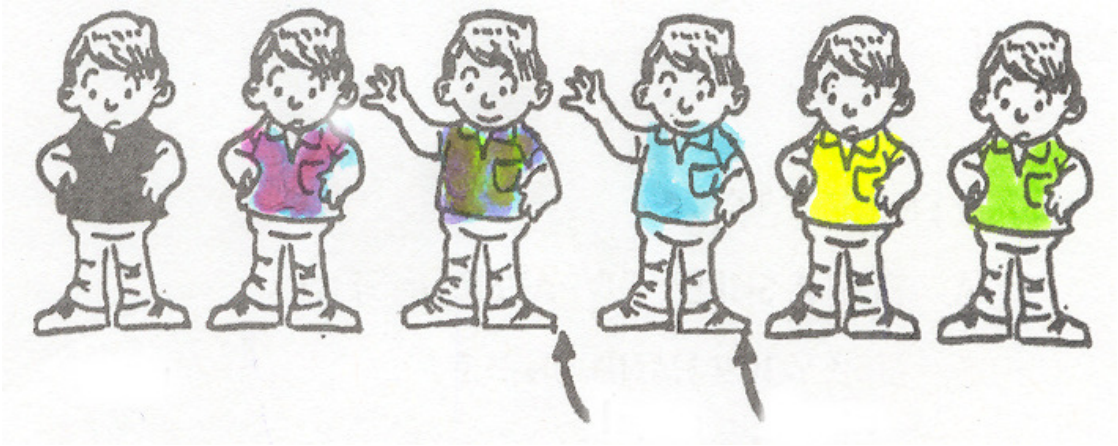
Two States

OFF ON

Switching State

COUNTER: 001100_{two}

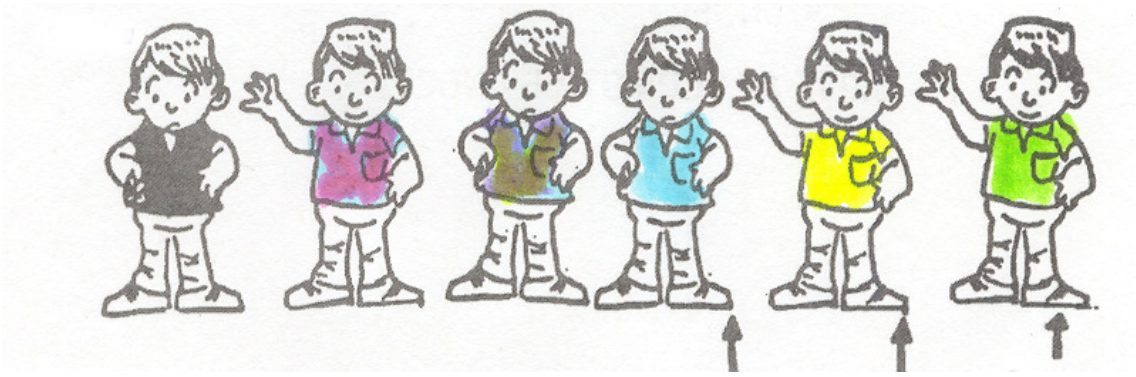
OVERFLOW



IMPULSE IMPULSE

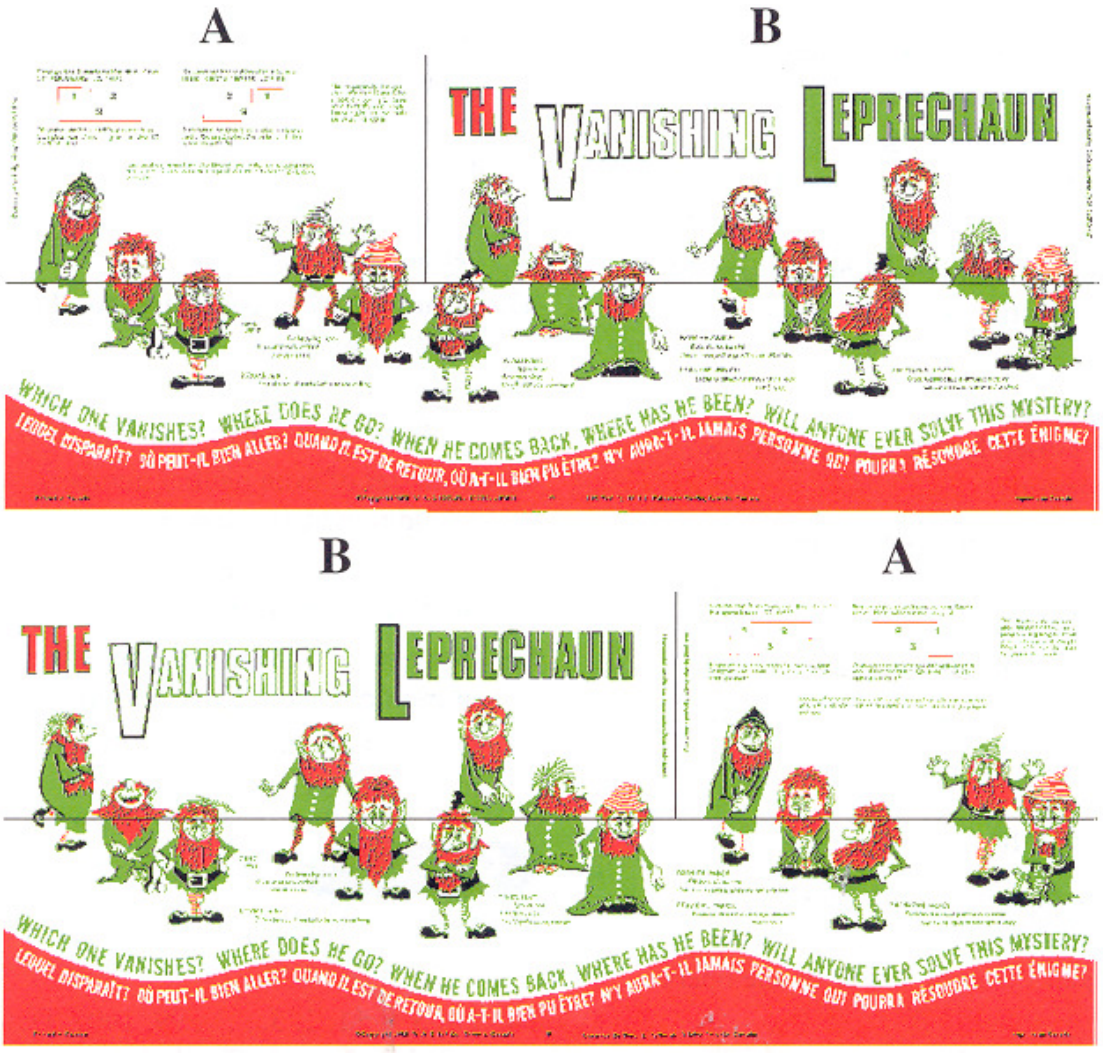
ADD: 111_{two}

OVERFLOW



IMPULSE IMPULSE IMPULSE

THE VANISHING LEPRECHAUN



I'VE BEEN WORKING ON THE

RAILROAD

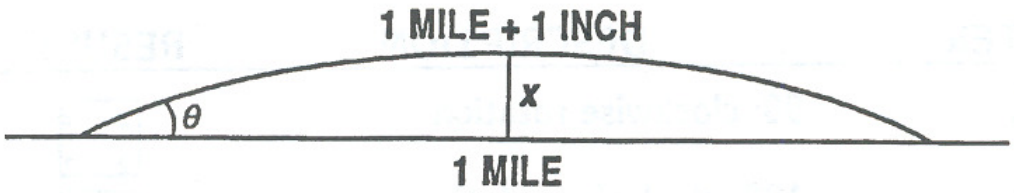
A single railroad track is laid one mile over level ground. It is firmly secured at the ends so that they cannot move. If in the heat of the day, the track expands one inch over its length and arcs up above the ground, then how high is the arc at its center?

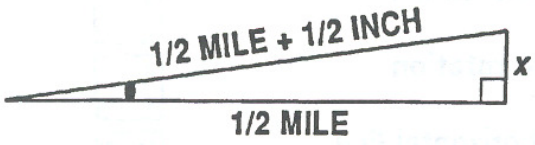
High enough to:

A. Slip a sheet of paper under?
B. Slip your hand under?
C. Crawl under?
D. Walk under?
E. Drive a locomotive under?

RAILROAD

PROBLEM SOLUTION





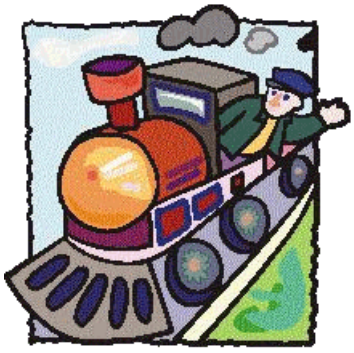
A triangle may be used to approximate x, since θ is so small (approx. 0.03°).

$$\left(\frac{1}{2}\text{mile} + \frac{1}{2}\text{inch}\right)^2 = x^2 + \left(\frac{1}{2}\text{ mile}\right)^2$$

$$(31{,}680.5)^2 = x^2 + (31{,}680)^2 \text{ in inches}$$

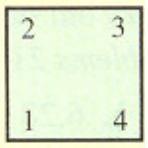
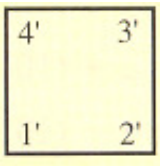
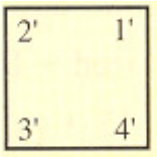
$$x = 177.989466 \text{ inches}$$

$$\approx 14.8 \text{ feet}$$



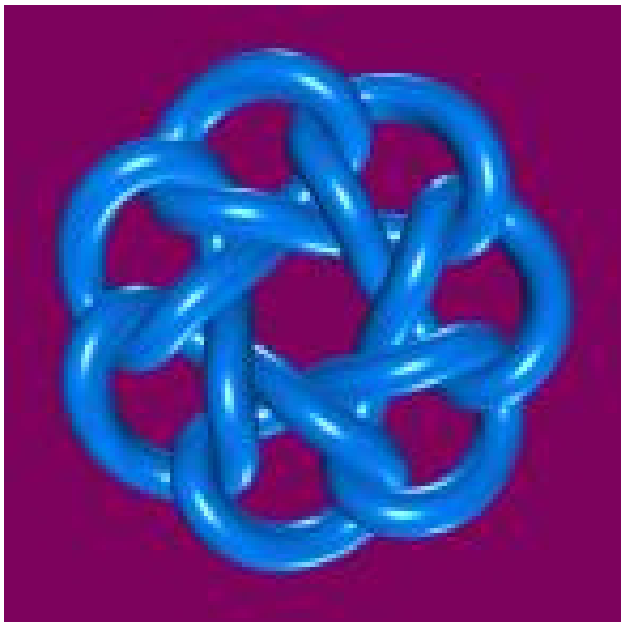
CASEY JONES CAN GO RIGHT UNDER!

SYMMETRIES OF A SQUARE

Element	*Description*	*Result*
A	**90°** clockwise rotation	
B	**180°** clockwise rotation	
C	**270°** clockwise rotation	
D	**360°** clockwise rotation	
E	Flip about a **horizontal** line through the middle of the square	
F	Flip about a **vertical** line through the middle of the square	
G	Flip along a line drawn from **upper** left to lower right	
H	Flip along a line drawn from **lower** left to upper right	

It's all up to YOU!

	A	B	C	D	E	F	G	H
A	B	C	D	A	H	G	E	F
B	C	D	A	B	F	E	H	G
C	D	A	B	C	G	H	F	E
D	A	B	C	D	E	F	G	H
E	G	F	H	E	D	B	A	C
F	H	E	G	F	B	D	C	A
G	F	H	E	G	C	A	D	B
H	E	G	F	H	A	C	B	D



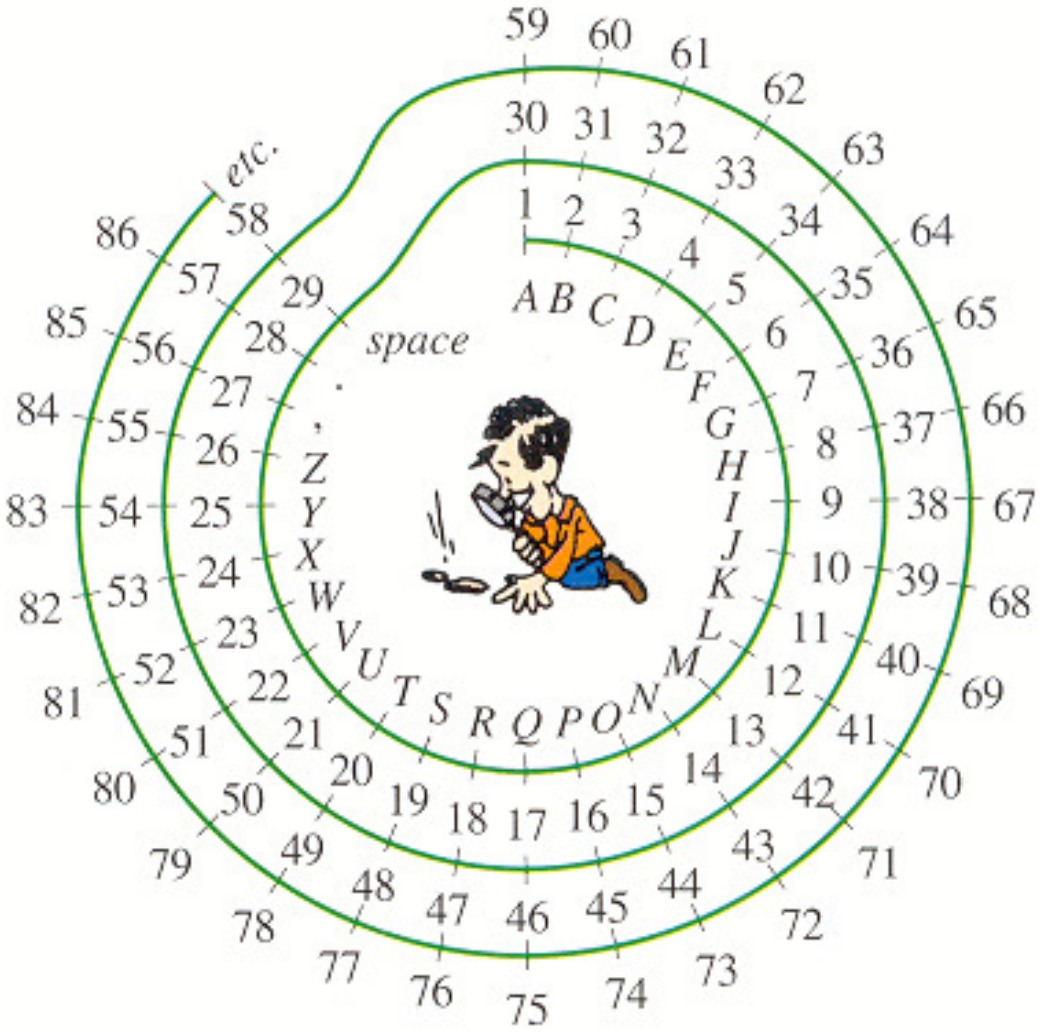
FIELD OF DREAMS

A **field** is a set $\mathbb{R}$, with two operations + and $\times$ satisfying the following properties for any elements a, b, $c \in \mathbb{R}$.

Addition	*Multiplication*
Closure	**Closure**
1. $(a + b) \in \mathbb{R}$	2. $ab \in \mathbb{R}$
Associative	**Associative**
3. $(a + b) + c = a + (b + c)$	4. $(a \times b) \times c = a \times (b \times c)$
Identity	**Identity**
5. There exists $0 \in \mathbb{R}$ so that $0 + a = a + 0 = a$ for every element a in $\mathbb{R}$.	6. There exists $1 \in \mathbb{R}$ so that $1 \times a = a \times 1 = a$ for every element a in $\mathbb{R}$.
Inverse	**Inverse**
7. For each $a \in \mathbb{R}$, there is a unique number $(-a) \in \mathbb{R}$ so that $a + (-a) = (-a) + a = 0$	8. For each $a \in \mathbb{R}$, $a \neq 0$, there is a unique number $\frac{1}{a} \in \mathbb{R}$ so that. $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$
Commutative	**Commutative**
9. $a + b = b + a$	10. $ab = ba$
Distributive	

11. $a \times (b + c) = a \times b + a \times c$

THE FBI HAS BED BUGS



Can you Unmultiply?

(More commonly known as FACTORING.)

PROCEDURE FOR FACTORING TRINOMIALS

Step 1 Find the factors of the second-degree term, and set up the binomials.

Step 2 Find the factors of the constant term, and consider all possible binomials (mentally). Think of the factors that will form a rectangle.

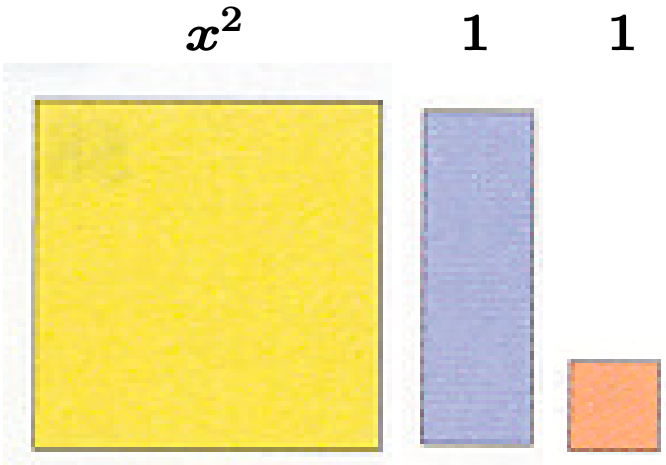
Step 3 Determine the factors that yield the correct middle term. If no pair of factors produces the correct full product, then the trinomial is not factorable using integers.

This factoring approach is called FOIL.

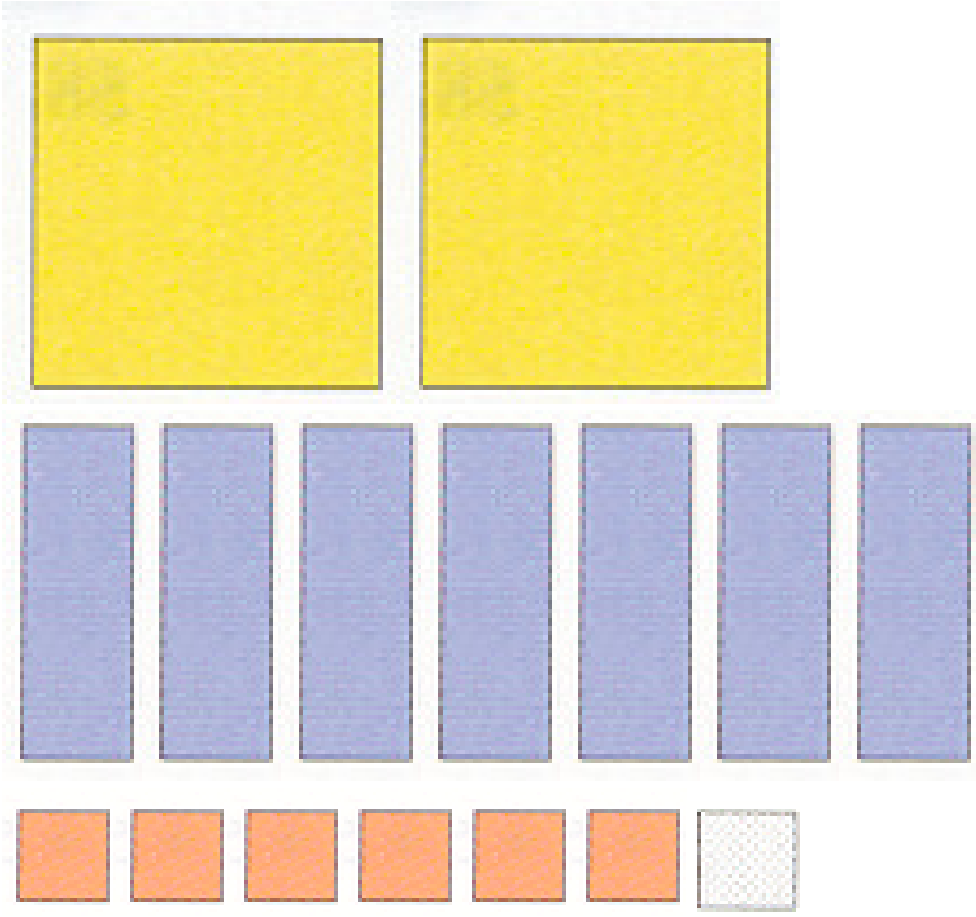
FACTORING USING

AREAS

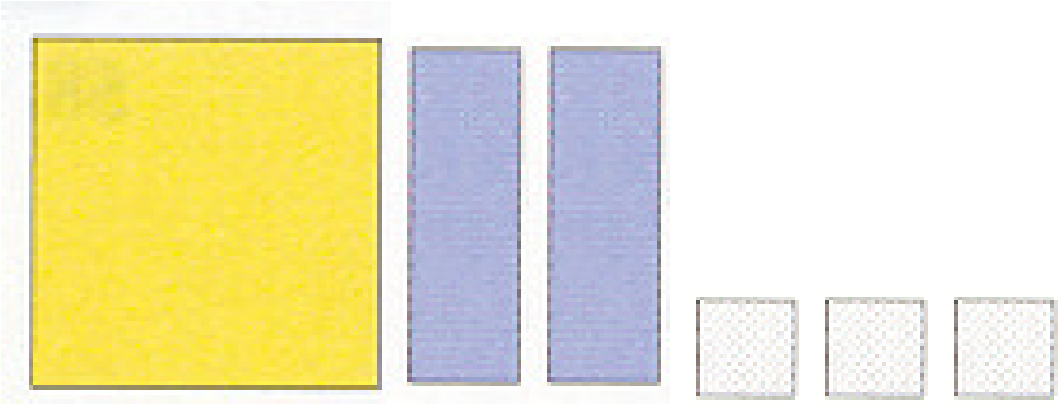
Area of a square with side x is x^2.
Area of a rectangle with sides x and 1 is x.
Area of a square with side 1 is 1.



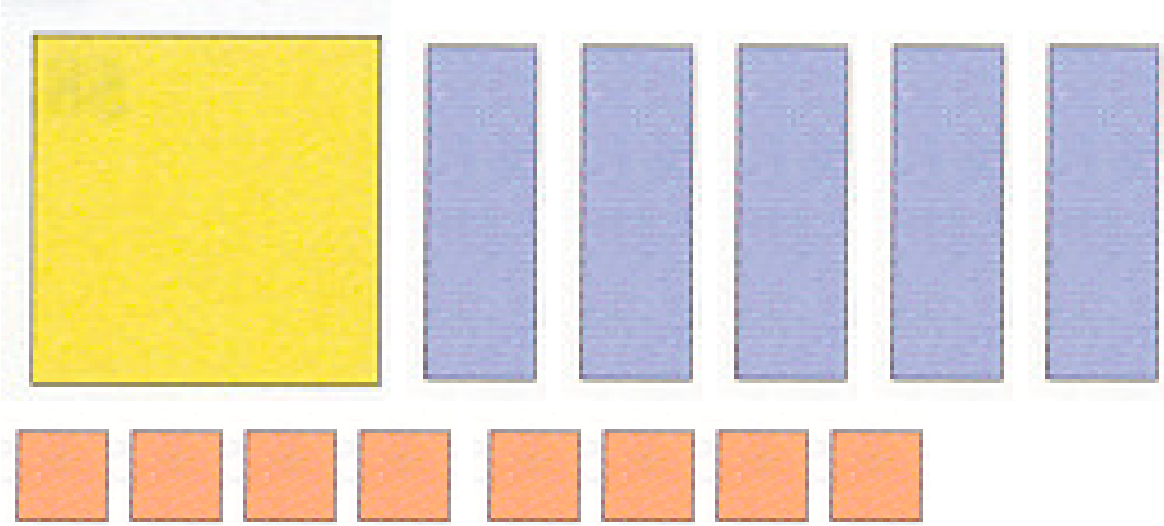
EXAMPLE 1:



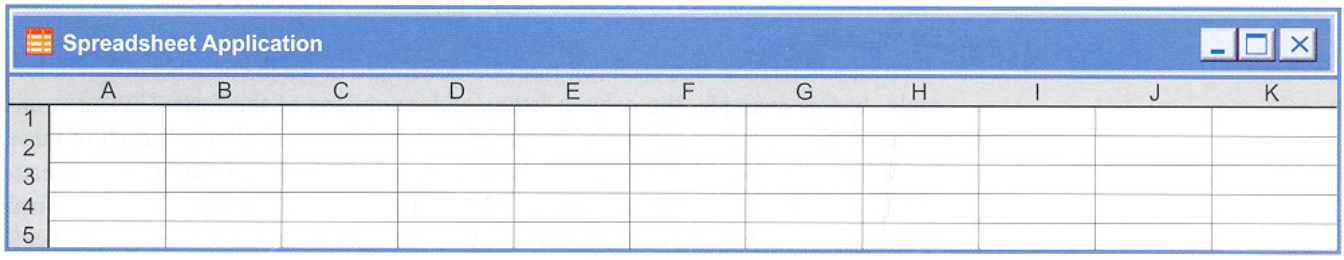
EXAMPLE 2a:

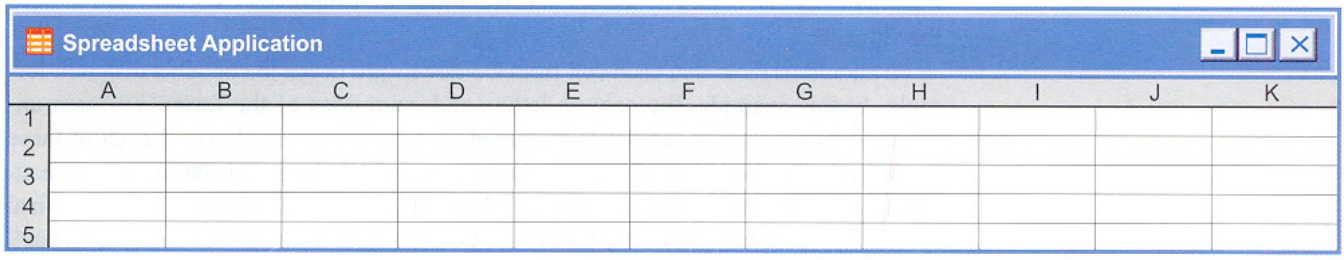


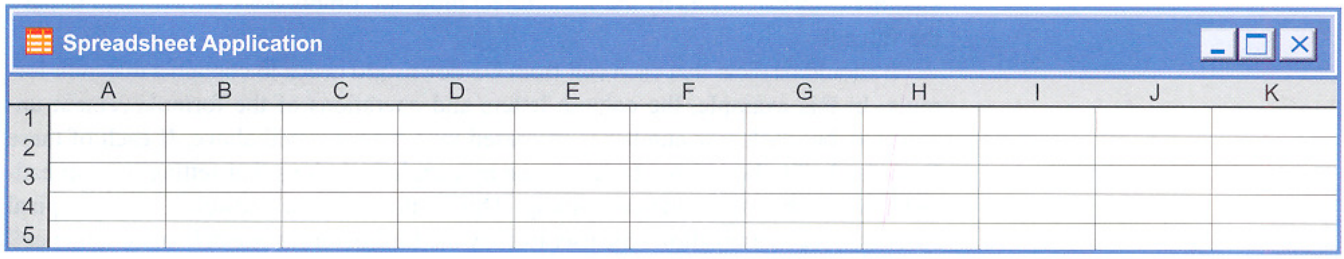
EXAMPLE 2b:

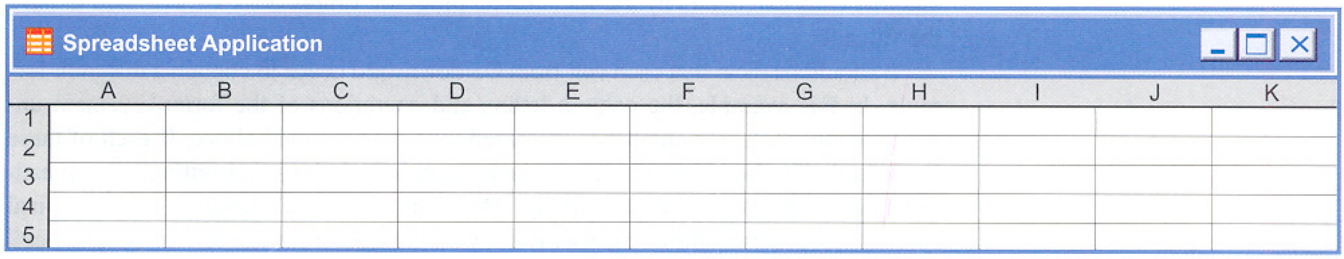


Sample Spreadsheets

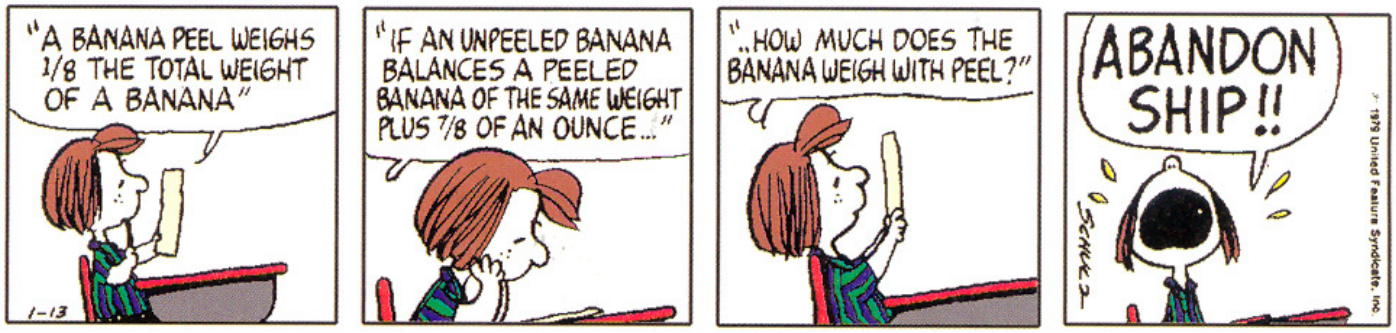








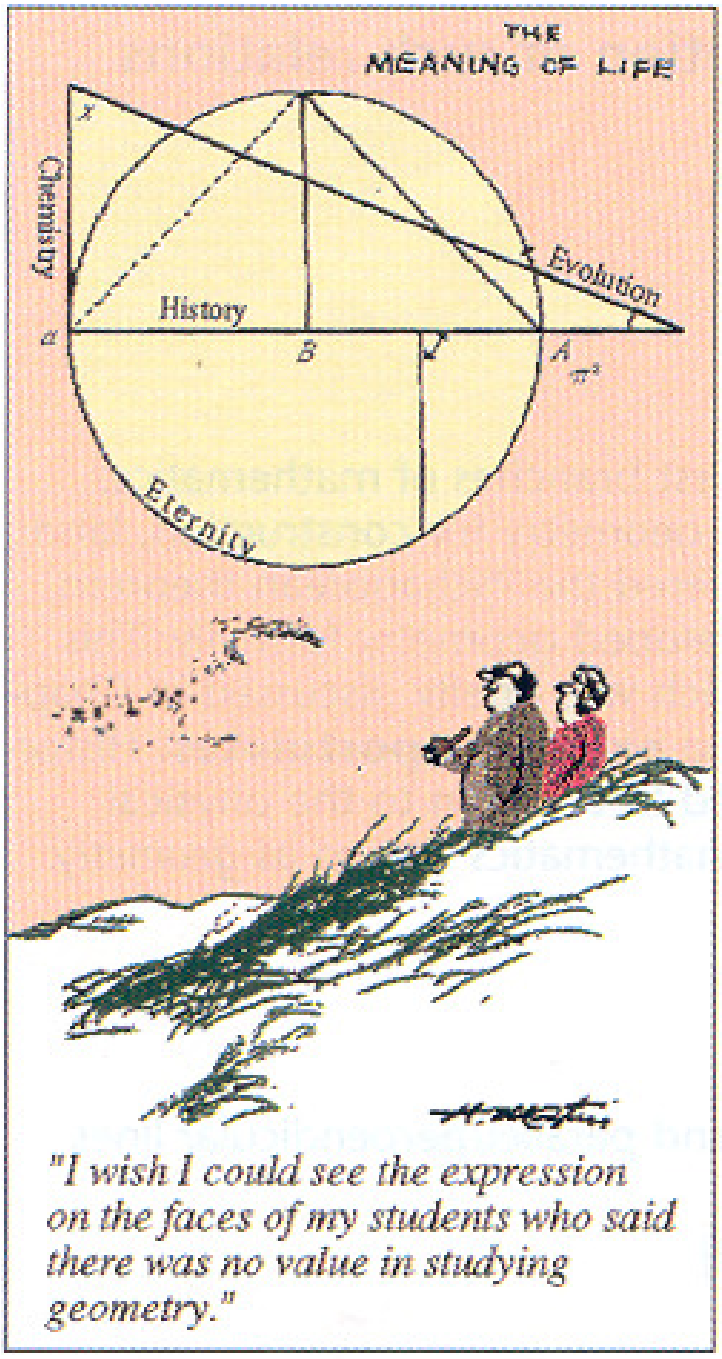
ABANDON SHIP!!!



Fractions, Decimals, and Percents

To / *From*	*Fraction*	*Decimal*	*Percent*
Fraction		Divide the numerator (top) by the denominator (bottom). Write as a terminating or as a repeating decimal (bar notation).	First change the fraction to a decimal by carrying out the division to two decimal places and writing the remainder as a fraction. *Then* move the decimal point two places to the right, and affix a percent symbol.
Terminating decimal	Write the decimal without the decimal point, and multiply by the decimal name of the last digit (rightmost digit).		Shift the decimal point two places to the *right*, and affix a percent symbol.
Percent	Write as a ratio to 100 and reduce the fraction. If the percent involves a decimal, first write the decimal in fractional form, and then multiply by $\frac{1}{100}$. If the percent involves a fraction, delete the percent symbol and multiply by $\frac{1}{100}$.	Shift the decimal point two places to the *left*, and delete the percent symbol. If the percent involves a fraction, first write the fraction as a decimal, and then shift the decimal point.	

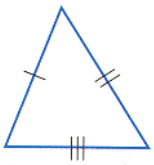
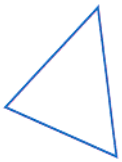
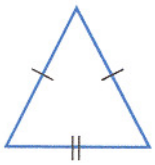
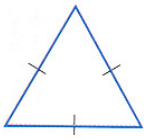
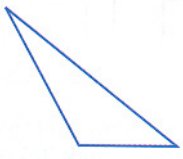
THE GRAND PLAN



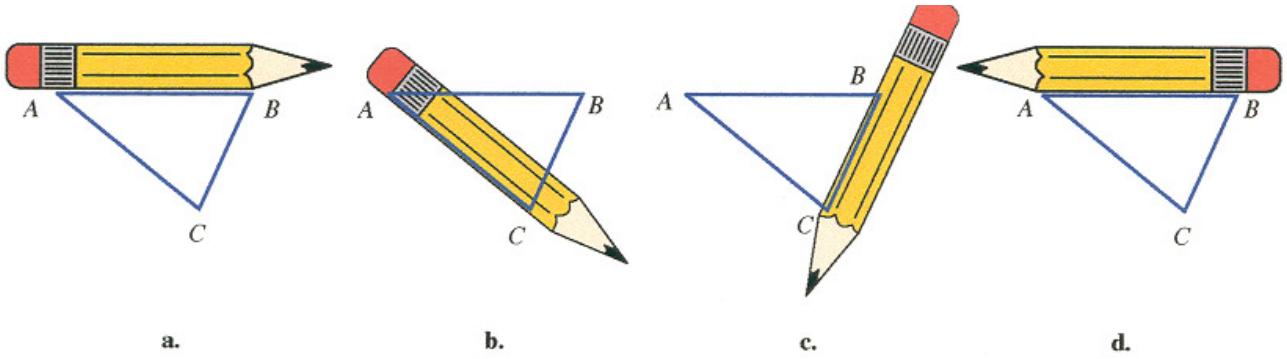
What do YOU see?

Anonymous, "Metamorphosis Landscape," Oil in Panel, 1600s

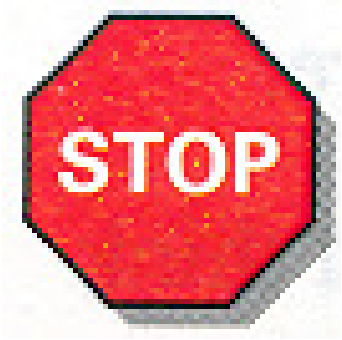
CLASSIFICATIONS OF TRIANGLES

By Sides	*By Angles*
Scalene: no equal sides	**Acute:** three acute angles
	
Scalene triangle	Acute triangle
Isosceles: two equal sides	**Right:** one right angle
	
Isosceles triangle	Right triangle
Equilateral: three equal sides	**Obtuse:** one obtuse angle
	
Equilateral triangle	Obtuse triangle

"AROUND" TRIANGLE



Demonstration that the sum of the measures of the angles in a triangle is 180°.



Remember....

The sum of the measures of the angles in ANY triangle is 180°.

Do you remember this?

PYTHAGOREAN THEOREM

For any right triangle with sides of lengths *a* and *b* and hypotenuse of length *c*

$$a^2 + b^2 = c^2$$

Also, if *a*, *b*, and *c* are the lengths of the sides of a triangle so that $a^2 + b^2 = c^2$ then the triangle is a right triangle.

HERE IS AN EXTENSION:

TRIGONOMETRIC RATIOS

In a right triangle *ABC* with right angle at *C*,

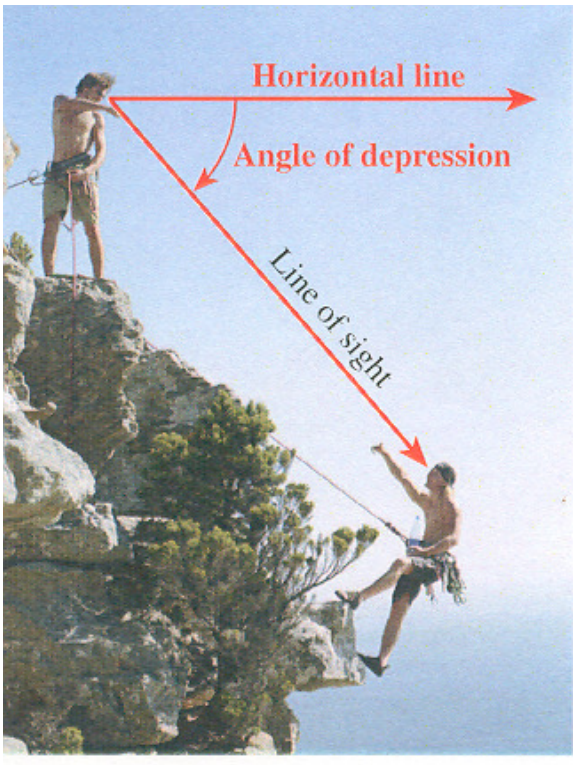
sin A is the ratio $\dfrac{\text{opposite side of } A}{\text{hypotenuse}}$

cos A is the ratio $\dfrac{\text{adjacent side of } A}{\text{hypotenuse}}$

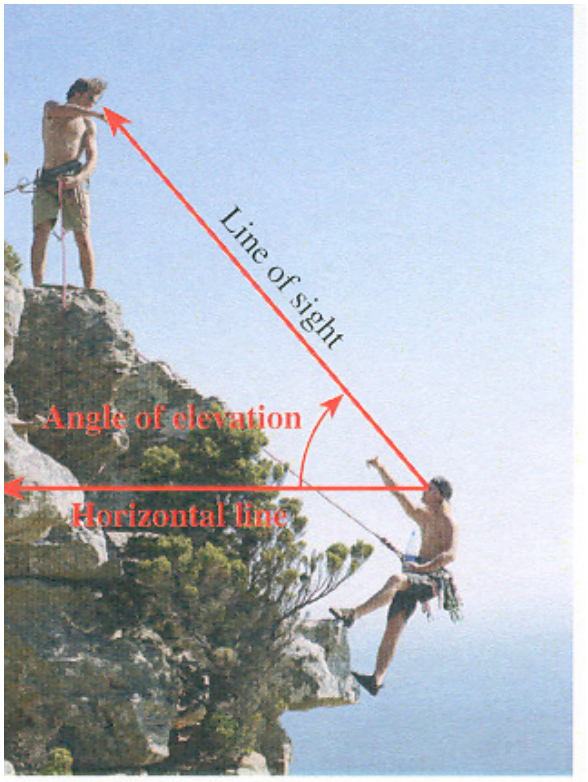
tan A is the ratio $\dfrac{\text{opposite side of } A}{\text{adjacent side of } A}$

IT'S IN THE EYE OF THE BEHOLDER

Bigshots/The Image Bank/Getty Images



angle of depression



angle of elevation

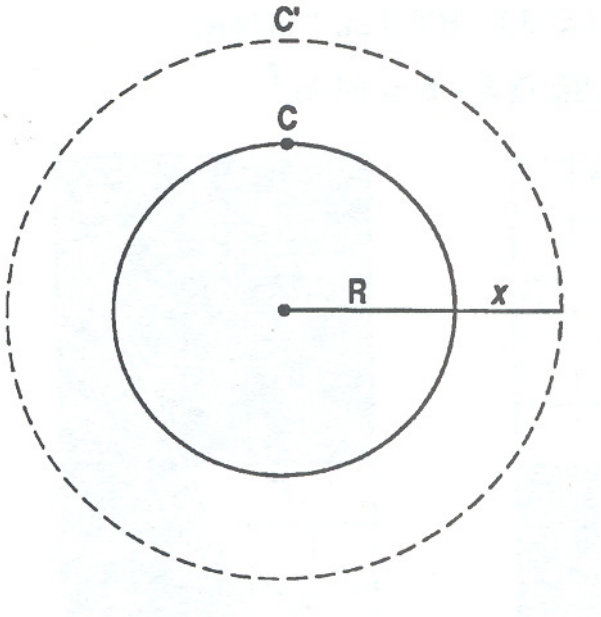
Transparency 35

STEEL BAND PROBLEM

Suppose that we fit a band tightly around the earth at the equator. We wish to raise the band so that it is uniformly supported 10 ft above the earth at the equator. Assuming that the slack is uniform all the way around the equator, the band would be loose enough to:

A. Walk under?
B. Crawl under?
C. Slip your hand under?
D. Slip a sheet of paper under?
E. Not even get the sheet of paper under?

STEEL BAND PROBLEM SOLUTION



$$
\begin{aligned}
C &= 2\pi R \\
C' &= 2\pi(R + x) \\
C' - C &= 2\pi(R + x) - 2\pi R \\
10 &= 2\pi R + 2\pi x - 2\pi R \\
10 &= 2\pi x \\
\frac{10}{2\pi} &= x \\
x &\approx 1.6
\end{aligned}
$$

The band would allow a uniform distance of 1.6 ft all the way around the equator. (ENUF TO CRAWL!)

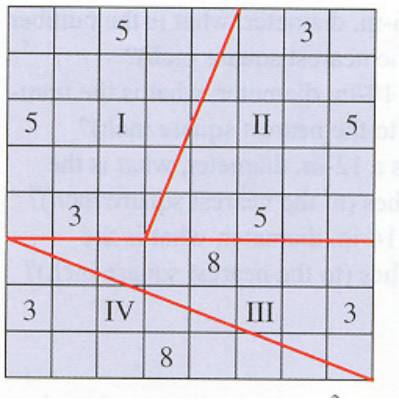
Moreover, wouldn't it be the same around the moon? Or a basketball?

Note the result is independent of R!!!

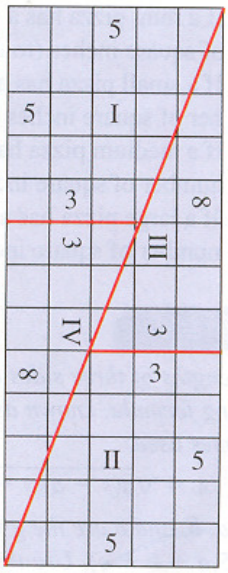
EXTRA SQUARE INCH

Consider the following 8 in. by 8 in. square.

The area of this square is $8 \times 8 = 64$ in.^{2}



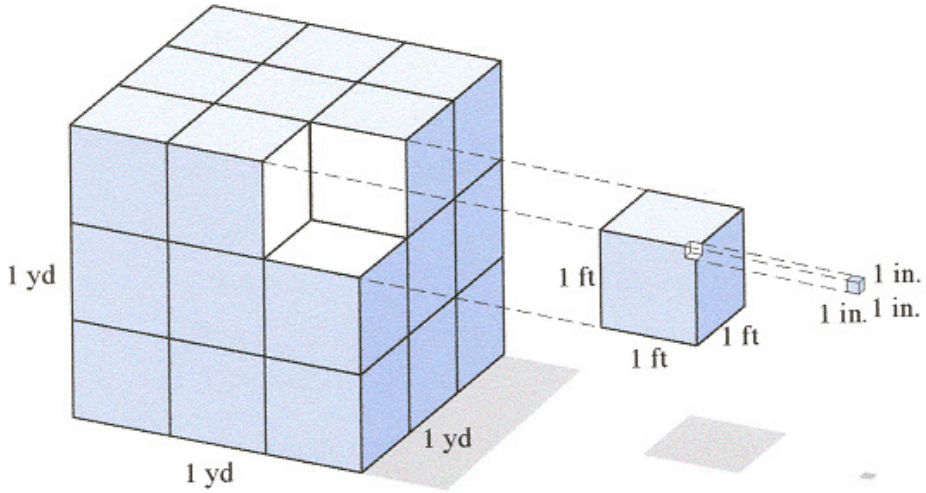
Cut this square into 4 pieces (I, II, III, IV) and rearrange the pieces as shown here. The area of this square is $5 \times 13 = 65$ in.^{2}



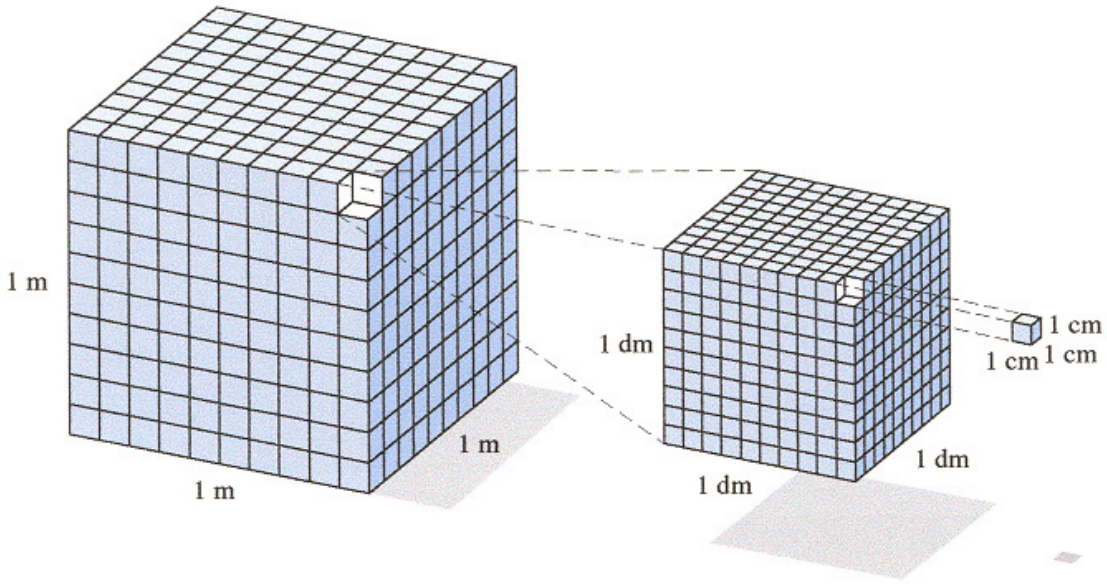
WHERE DID THE EXTRA SQUARE INCH COME FROM?

VOLUME AND CAPACITY

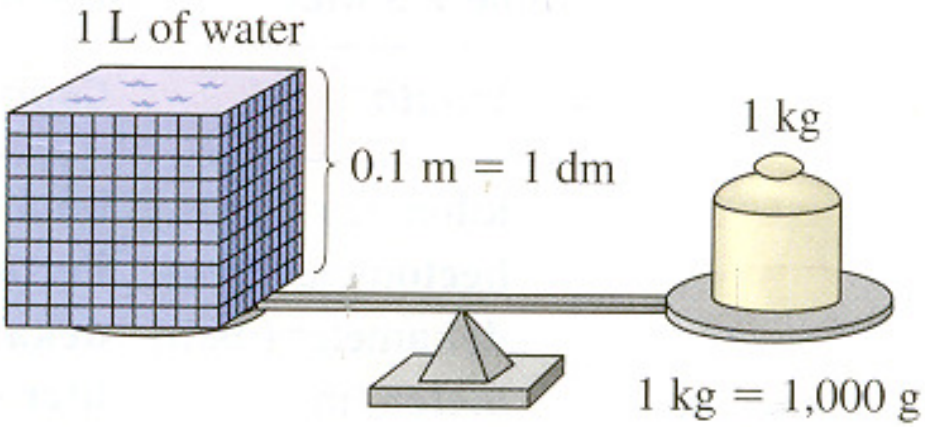
1 $yd^3 \approx 200$ gal, 1 $ft^3 \approx 7.5$ gal, 1 gallon $\approx$ 231 $in.^3$



1 m^3 = 1,000 L, 1 dm^3 = 1 L, 1 cm^3 = 1 cc = 1 mL



METRIC SYSTEM



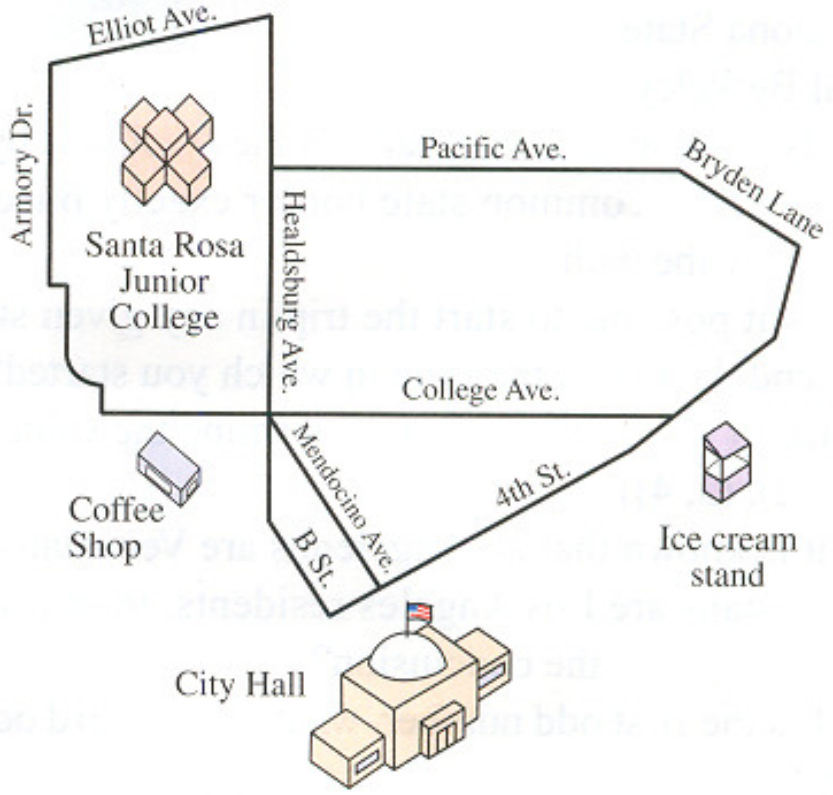
Length: meter

Capacity: liter

Weight: gram

Santa Rosa Street Problem

On Saturday evenings, a favorite pastime of students is the cruise the marked streets in Santa Rosa.



Is it possible to choose a route so that all the permitted streets are traveled exactly once?

The intersections are identified by the following buildings:

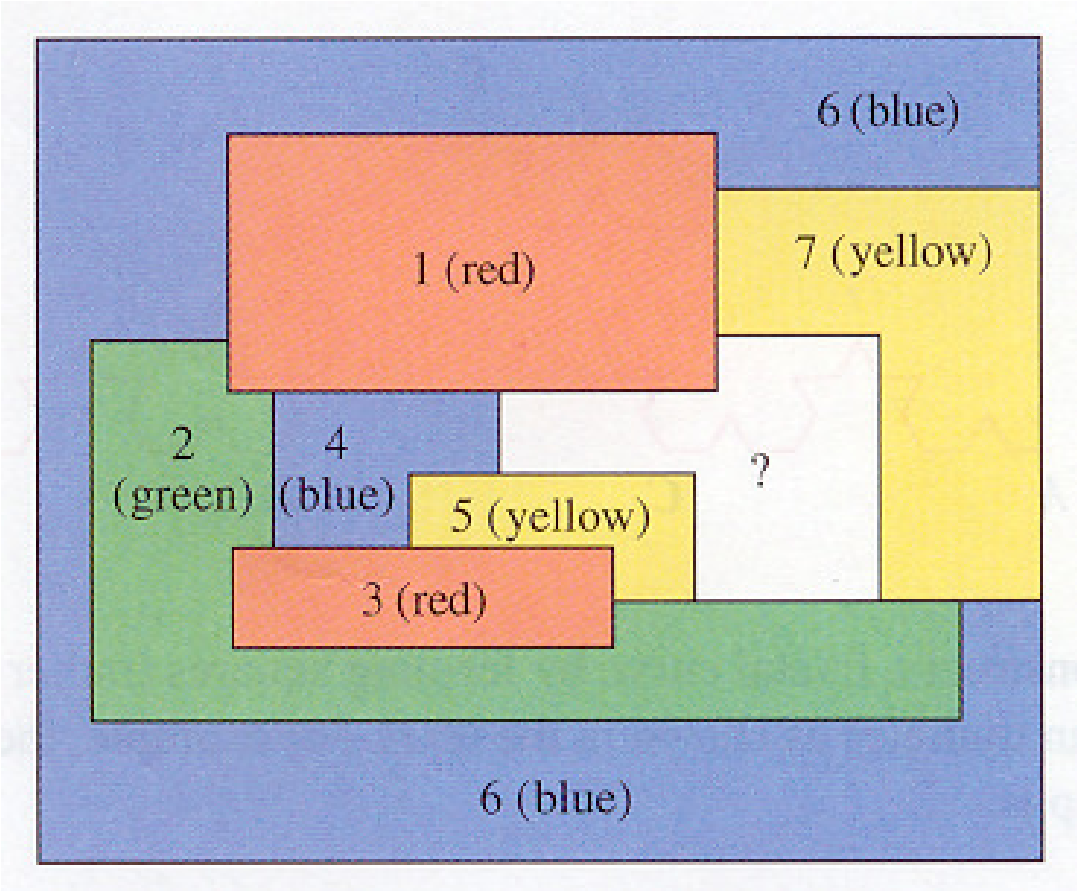
SRJC (Mendocino and Pacific Avenues)

Coffee Shop (Mendocino, College, and Healdsburg Avenues)

City Hall (Mendocino and Fourth Streets)

Fresh Freeze (College and Fourth Streets)

Is this the world's FIRST FIVE-COLOR MAP?



What color should you use for the region marked "?"?

Note that this region is bounded by each of the four colors.

What is a logarithm?

How do you pronounce $b^x = N$?

x is the exponent on a base b that gives the answer N.

or

x = the exp on a base b to get N

or

$x = \exp_b N$

Remember, $x = \exp_b N$ is

"x is the exponent on a base b that gives N"

Write "log" to mean "exponent" or "exp", That is,

$$x = \log_b N$$

is

"x is the exponent on a base b that gives N"

SHROUD OF TURIN

In 1988 ^{14}C testing showed 92.3% of original remained.

I. Pilon/Shutterstock

$$A = A_0 e^{rt}$$

TIME IS MONEY

Financial Variables

- P for present value (or principal)
- A for present value (or principal)
- I for interest
- r for annual percentage rate
- t for time (in years)
- m for amount of periodic payment
- n for the number of times compounded per year

Compound Interest

Find the future value for \$1 invested at 100% interest for 1 year; *i.e.,* Find A for $P = 1, r = 1, t = 1$.

Number of Periods	Formula	Amount
Annual, $n = 1$	$\left(1 + \frac{1}{1}\right)^1$	\$2.00
Semiannual, $n = 2$	$\left(1 + \frac{1}{2}\right)^2$	\$2.25
Quarterly, $n = 4$	$\left(1 + \frac{1}{4}\right)^4$	\$2.44
Monthly, $n = 12$	$\left(1 + \frac{1}{12}\right)^{12}$	\$2.61
Daily, $n = 360$	$\left(1 + \frac{1}{360}\right)^{360}$	\$2.71

Define $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \approx 2.718281828$

ARE YOU A GENIUS?

Problem 1

A, D, G, J, . . .

Problem 2

1, 3, 6, 10, . . .

Problem 3

1, 1, 2, 3, 5, . . .

Problem 4

21, 20, 18, 15, 11, . . .

Problem 5

8, 6, 7, 5, 6, 4, . . .

Problem 6

40, 35, 34, 29, 28, 23, . . .

From *Mensa* test. Reprinted by permission by Mensa, 50 E. 42 St., New York, NY 10017

ELEVEN PUZZLE

Pick any two integers between -5 and 5.
Add these two numbers to fill in space #3.
Add #2 and #3 to fill in #4.
Continue until you have filled in 10 numbers.
What is the sum of these 10 numbers?

Pick any two numbers:	**(1)** _____	n
	(2) _____	m
Add to obtain a third:	**(3)** _____	$n+m$
Continue:	**(4)** _____	$n+2m$
	(5) _____	$2n+3m$
	(6) _____	$3n+5m$
	(7) _____	$5n+8m$
	(8) _____	$8n+13m$
	(9) _____	$13n+21m$
	(10) _____	$21n+34m$
Add the entire column:	_____	$55n+88m$

Fibonacci's Delight

Notice: $55n+88m=11(5n+8m)$.
This is 11(7th number).

WARNING!!

The wrong answer to this question could cost you $4,000/mo for the rest of your life!

From the Chapter Overview:

The stated goal of this book is to strengthen your ability to solve problems — not the classroom type of problems, but those problems that you may encounter as an employee, a manager, or in everyday living. You can apply your problem-solving ability to your financial life. A goal of this chapter might well be to put some money into your bank account that you would not have had if you had not read this chapter.

As a preview to this chapter, consider the question asked in Example 2 of Section 11.5 (page 544):

Suppose you are 21 years old and will make monthly deposits to a bank account paying 10% annual interest compounded monthly. Which is the better option?

Option I: Pay yourself $200 per month for 5 years and then leave the balance in the bank until age 65. (Total amount of deposits is $\$200 \times 5 \times 12 = \$12{,}000$.)

Option II: Wait until you are 40 years old (the age most of us start thinking seriously about retirement) and then deposit $200 per month until age 65. (Total amount of deposits is $\$200 \times 25 \times 12 = \$60{,}000$.)

Obtaining a Home Loan

Home loans are quoted by giving RATE (APR), POINTS, and FEES.

Here is a sample:

Bank of America: 6.23%, 2 pts, $400
Central Bank: 6.8%, 0 pts, $200
River City: 6.3%, 1.5 pts, no fee
City Bank: 5.5%, 4 pts, $300
First Interstate: 6.28%, 1 pt, $150

Which is the best offer? To compare different loan offers, you can use the following formula:

$$\text{COMPARISON RATE} = \text{APR} + 0.125\left(\text{POINTS} + \frac{\text{FEES}}{\text{AMOUNT OF THE LOAN}}\right)$$

You can also write a spreadsheet program to do this calculation.

	A	B	C	D	E	F
1	**Comparison Rates for Home Loans**					
2	**What is the amount of the loan?**			$100,000.00		
3						
4	Bank	APR	POINTS	FEES	COMPARISON RATE	
5	Bank of America	6.23%	2	$400	6.53%	
6	Central Bank	6.80%	0	$200	6.83%	
7	River City	6.30%	1.5	$0	6.49%	
8	City Bank	5.50%	4	$300	6.04%	
9	First Interstate	6.28%	1	$150	6.42%	
10						

City Bank is the best offer.

CHANTERELLE

LUNCH

Prix fixe

October 3rd – 29th, 2005

Appetizers

The Soup of the Day

or

The Composed Salad of the Day

or

Classic Terrine of Muscovy Duck & Foie Gras

~

Entrées

Breast of Free Range Organic Chicken with Preserved Lemon & Greek Olives

or

Striped Bass with Red Wine and Fresh Sage

or

The Entrée of the Day

~

Desserts

An Assortment of Sherbets

or

Warm Crispy Hazelnut Streusel Topped Pumpkin & Hazelnut Tart with Maple Ginger Ice Cream

Coffee or Tea

Chocolate Truffles

$42.00

Courtesy of Chanterelle Restaurant, New York.

We the people. . .

The personalized license plates were done by Stephen Underwood, who not only assembled the plates to spell out the preamble of the U.S. Constitution, but also did it by using all 50 states *in alphabetical order*!

Smithsonian Art Museum, Washington, D.C./Art Resource, NY.

EXPERIMENT 1
Rolling One Die

Name of Team _____

_____

Directions: One team member rolls a single die 50 times. The other person records the outcomes using tally marks on the table below. For Trial 2, switch places and repeat the experiment.

OUTCOME	TRIAL 1 Frequency	TRIAL 2 Frequency	TOTAL	PERCENT
1				
2				
3				
4				
5				
6				

Total the tally marks for Trials 1 and 2.
Next, calculate the percentage of occurrence (divide the total number of times for Outcome 1 by 100 to find the percentage of occurrence for Outcome 1).

EXPERIMENT 2 Rolling Two Dice

Directions: One team member rolls a pair of dice 50 times. The other person records the outcomes using tally marks on the table below. For Trial 2, switch places and repeat the experiment.

OUTCOME	TRIAL 1 Frequency	TRIAL 2 Frequency	TOTAL	PERCENT
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				

Total the tally marks for Trials 1 and 2.
Next, calculate the percentage of occurrence.

EXPERIMENT 3 A Coin and A Die

Directions: One team member simultaneously tosses a coin and rolls a since die 50 times. The other person records the outcomes using tally marks on the table below. For Trial 2, switch places and repeat the experiment.

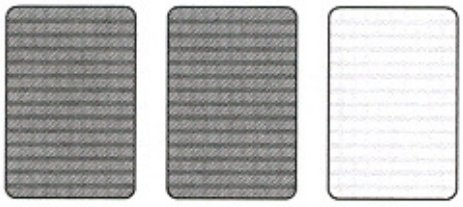
OUTCOME	TRIAL 1 Frequency	TRIAL 2 Frequency	TOTAL	PERCENT
1H				
1T				
2H				
2T				
3H				
3T				
4H				
4T				
5H				
5T				
6H				
6T				

Total the tally marks for Trials 1 and 2.
Next, calculate the percentage of occurrence.

How does P(3H) compare with P(3) and P(3)?
How does P(even number or 3) compare with P(even) and P(3)?
How does P(even number or head) compare with P(even) and P(head)?
Make a conjecture about P(E or F)

EXPERIMENT 4 Three Card Problem

Directions: You need to prepare three 3×5 cars so that they are indistinguishable expect for the color.



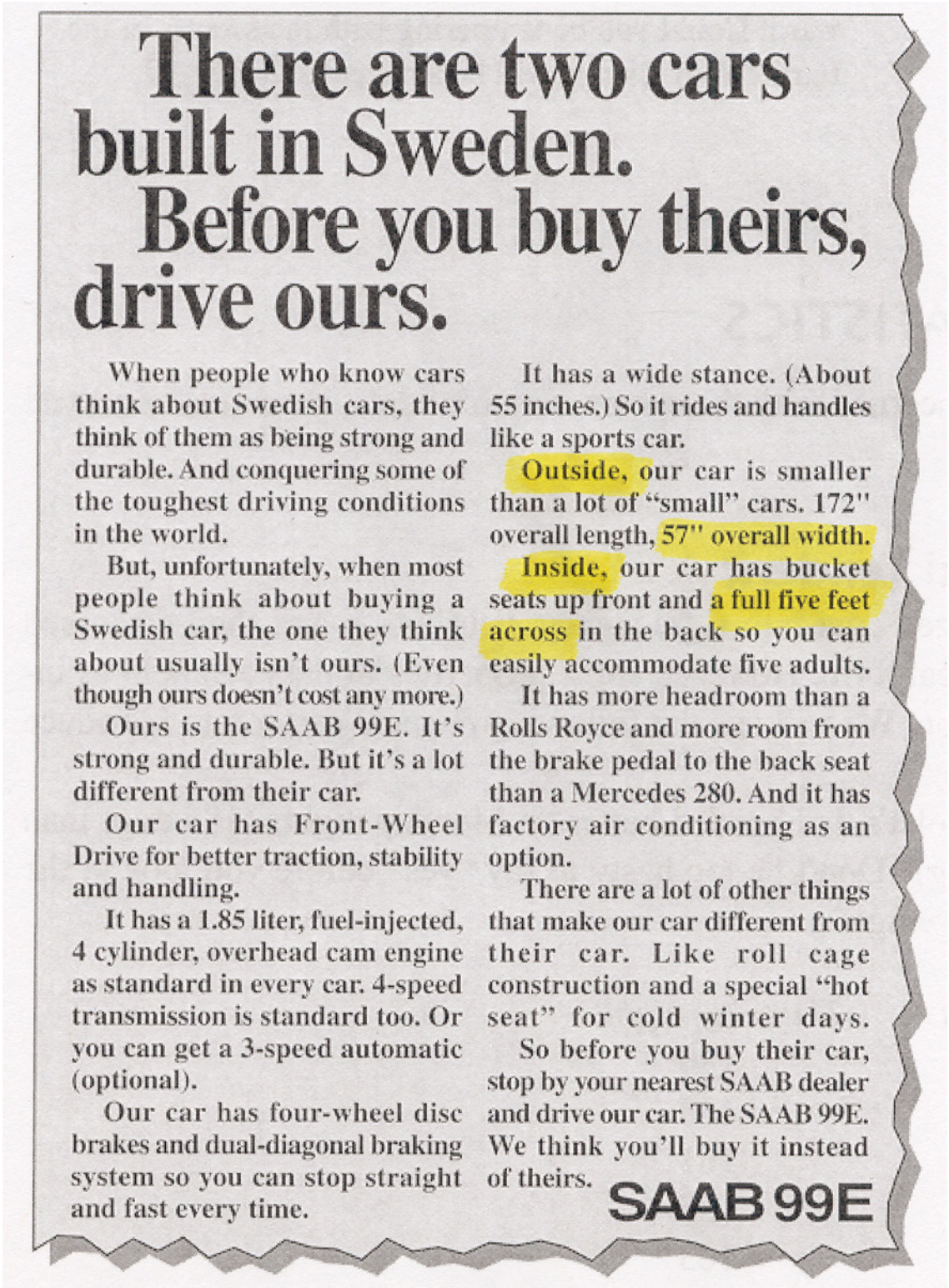
Black on both sides | Black on one side, white on the other | White on both sides

One team member "shuffles" the three cards under a table so that the cards cannot be seen. Be sure to flip some cards over and back and forth so you don't know which side is "up" or which card is on top. Now, select one card at random and place it flat on a table — be careful not to look at the bottom of this card. You will see either a black or white card. Record the color in Column A. This is not the probability with which we are concerned. Rather, we are interested in predicting the probability of the *other* side being black or white. Record the color of the second side in Column B. Repeat the experiment 100 times.

Top	COLUMN A	OUTCOME Bottom	COLUMN B	COLUMN C
WHITE		WHITE		
		BLACK		
BLACK		WHITE		
		BLACK		

The total number of tally marks in Column A should be 100. Find the percentage of occurrence in Column C. To find the percentage of white/white, divide your entry in Column B (white) by the entry in Column A (white). To find the percentage of white/black, divide your entry in Column B (black) by the entry in Column A (white). These two percentages should add up to 100%. Now, do the same to find the percentage of black/white and black/black. Are these the results you expected? Why or why not?

CAVEAT EMPTOR



There are two cars built in Sweden. Before you buy theirs, drive ours.

When people who know cars think about Swedish cars, they think of them as being strong and durable. And conquering some of the toughest driving conditions in the world.

But, unfortunately, when most people think about buying a Swedish car, the one they think about usually isn't ours. (Even though ours doesn't cost any more.)

Ours is the SAAB 99E. It's strong and durable. But it's a lot different from their car.

Our car has Front-Wheel Drive for better traction, stability and handling.

It has a 1.85 liter, fuel-injected, 4 cylinder, overhead cam engine as standard in every car. 4-speed transmission is standard too. Or you can get a 3-speed automatic (optional).

Our car has four-wheel disc brakes and dual-diagonal braking system so you can stop straight and fast every time.

It has a wide stance. (About 55 inches.) So it rides and handles like a sports car.

Outside, our car is smaller than a lot of "small" cars. 172" overall length, 57" overall width.

Inside, our car has bucket seats up front and a full five feet across in the back so you can easily accommodate five adults.

It has more headroom than a Rolls Royce and more room from the brake pedal to the back seat than a Mercedes 280. And it has factory air conditioning as an option.

There are a lot of other things that make our car different from their car. Like roll cage construction and a special "hot seat" for cold winter days.

So before you buy their car, stop by your nearest SAAB dealer and drive our car. The SAAB 99E. We think you'll buy it instead of theirs.

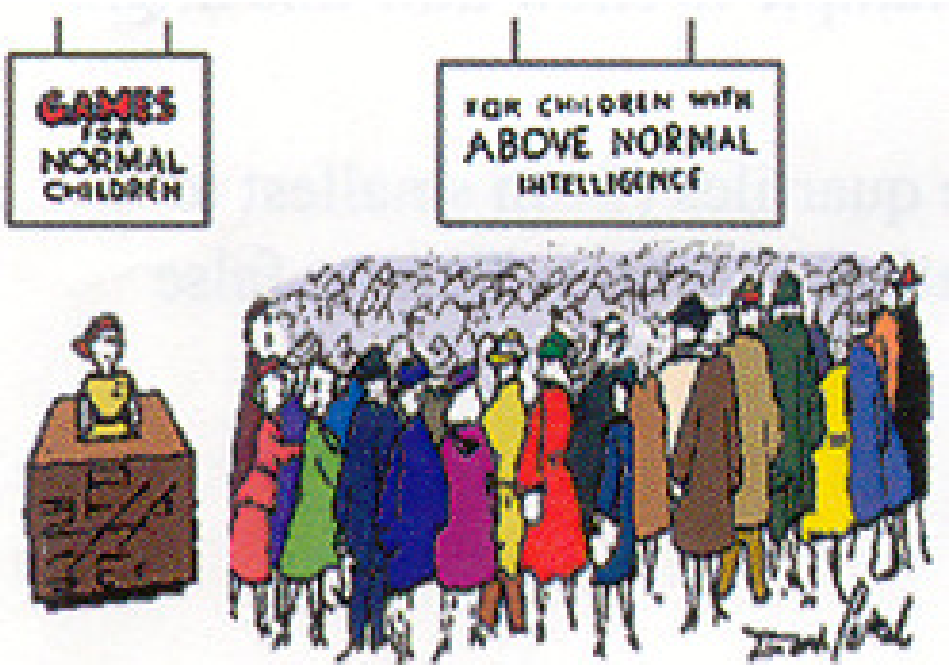
SAAB 99E

"My Dad is Better than Your Dad"

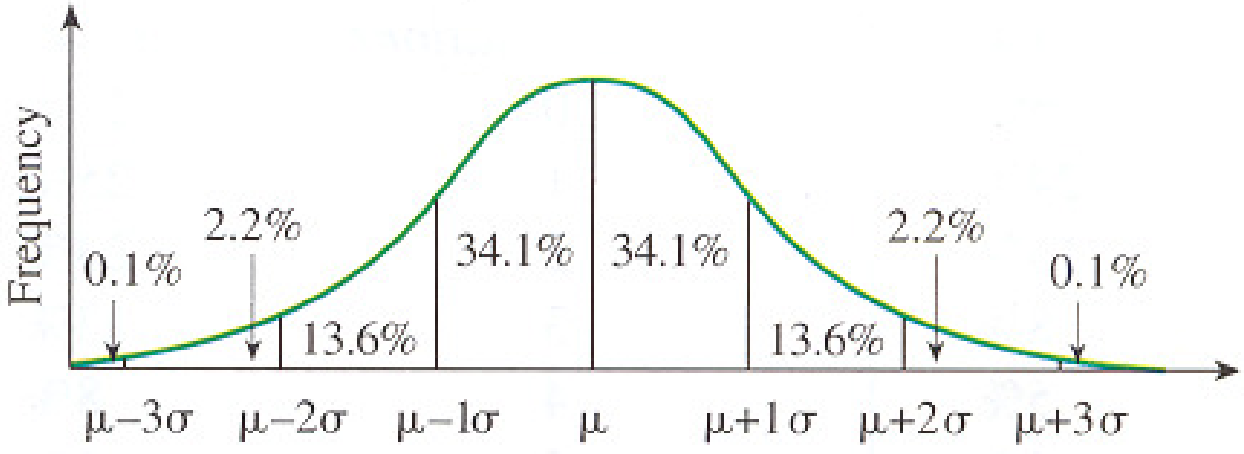
Do you suppose that Violet's dad bowled better on Monday nights (185 avg) than on Thursday nights (170 avg)? Don't be too hasty to say "yes" before you look at the scores that make up these averages:

	Monday night	*Thursday night*
Game 1	175	180
Game 2	150	130
Game 3	160	161
Game 4	180	185
Game 5	160	163
Game 6	183	185
Game 7	287	186
Totals	1,295	1,190

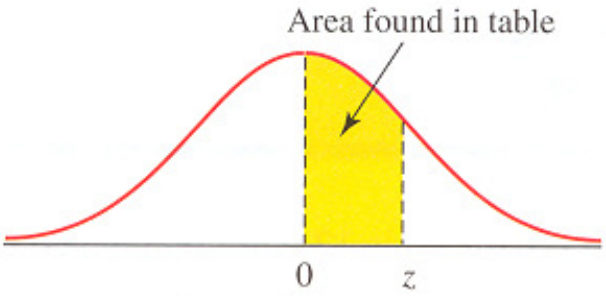
Are you normal?



Cartoon by David Pascal



Z-scores



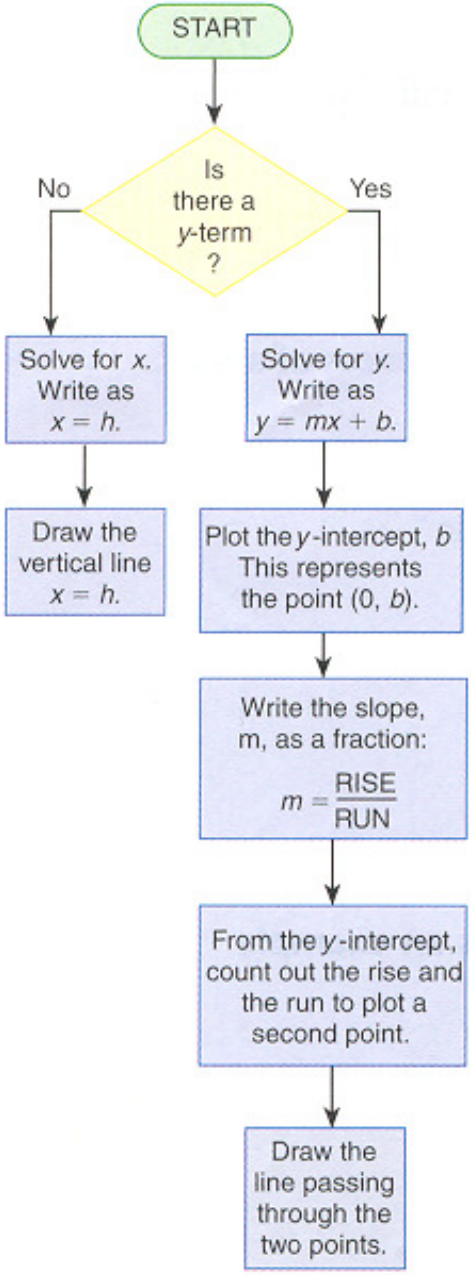
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Note: For values of z above 3.09, use 0.4999.

Linear Equations

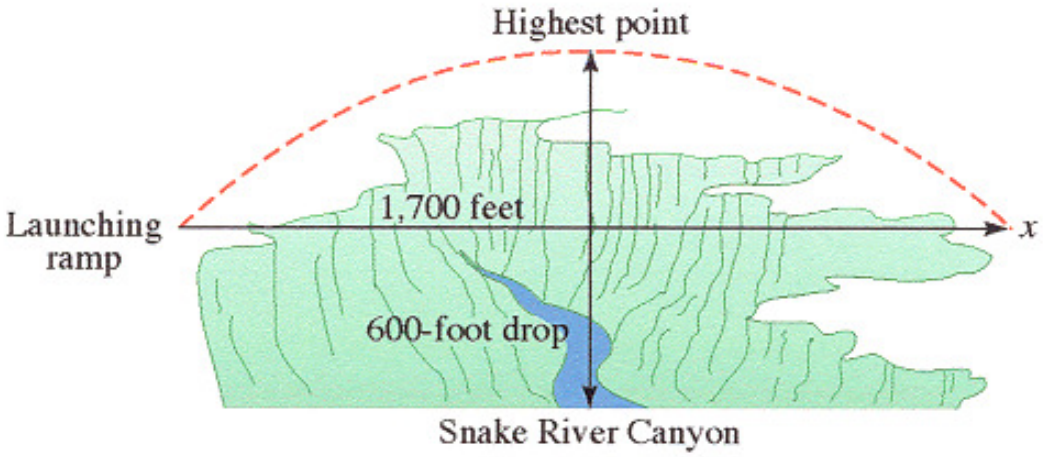
STANDARD FORM: $Ax + By + C = 0$
POINT-SLOPE FORM: $y = mx + b$

Procedure for Graphing



Will Evel make it?

"Well, Billy, can I make it?" asked Evel. "If you can accelerate to the proper speed, and if the wind is not blowing too much, I think you can," answered Billy. "It will be one huge money-maker, but I want some assurance that it can be done!" retorted Evel. About 30 years ago, a daredevil named Evel Knievel attempted a Skycycle ride across the Snake River.

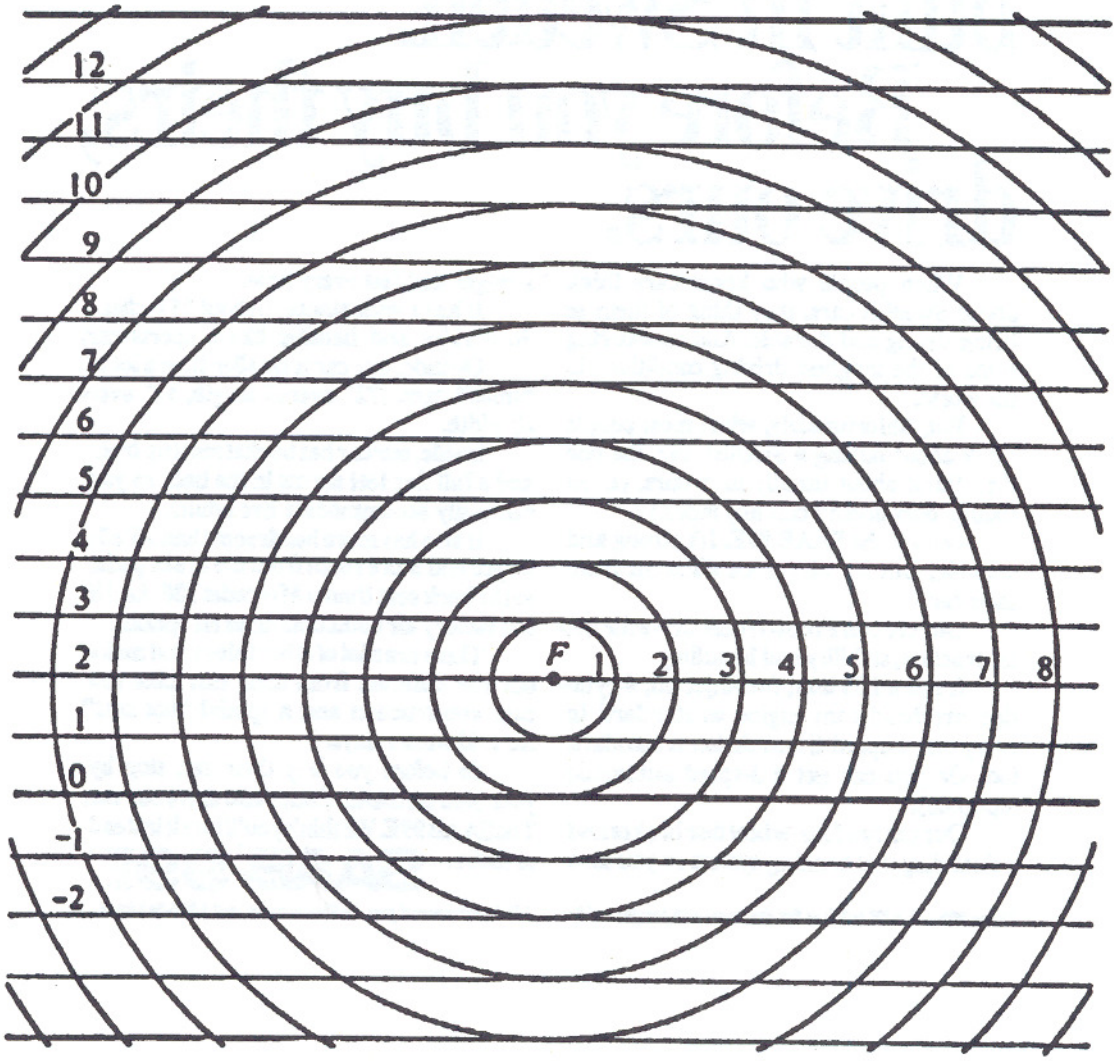


Suppose that the path of Evel Knievel's Skycycle is

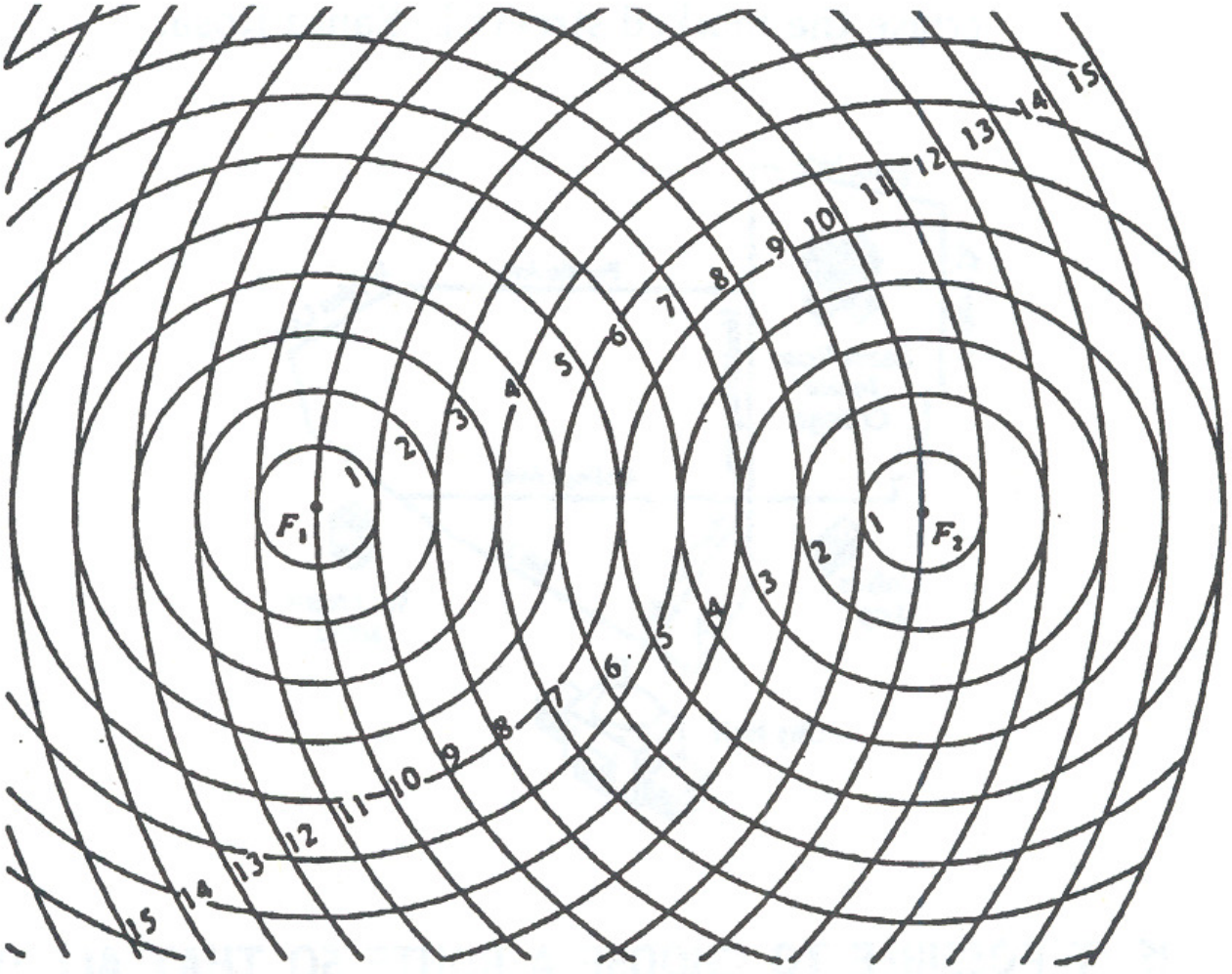
$$y = -0.0005x^2 + 2.39x$$

Assuming that the ramp is at the origin and x is the horizontal distance traveled, graph this relationship. Using your graph, answer Evel's question: Will he make it? Assume that the actual distance the Skycycle must travel is 4,700 ft.

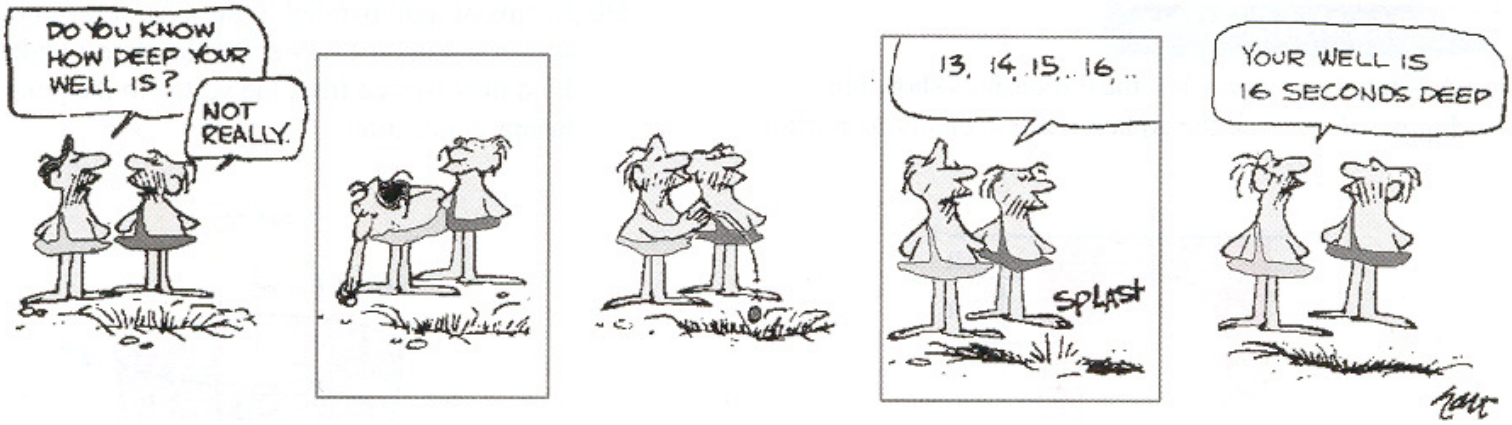
Parabola Graph Paper



Ellipse/Hyperbola Graph Paper



How Deep is Your Well?



A **function** is a set of ordered pairs in which the first component is associated with exactly one second component.

whereas f represents the *function*, $f(x)$ represents a *number*.

IT'S ELEMENTARY...

There are four *elementary row operations* for producing equivalent matrices:

1. **RowSwap**
2. **Row +**
3. ***Row**
4. ***Row +** 

This operation changes only the target row.

PIVOTING

A process known as pivoting means to carry out the following two steps.

Step 1 Divide all entries in the row in which the pivot appears (called the pivoting row) by the nonzero pivot element so that the pivot entry becomes a 1. This used elementary row operation *Row.

Step 2 Obtain zeros above and below the pivot element by using elementary row operation *Row + .

GAUSS-JORDAN ELIMINATION

Step 1 Select as the first pivot the element in the first row, first column, and pivot.

Step 2 The next pivot is the element in the second row, second column; pivot.

Step 3 Repeat the process until you arrive at the last row, or until the pivot element is a zero. If it is a zero and you can interchange that row with a row below it, so that the pivot element is no longer a zero, do so and continue. If it is zero and you cannot interchange rows so that it is not a zero, continue with the next row.

The final matrix is called the **row-reduced form.**

Maximizing Profit

A farmer has 100 acres on which to plant two crops, corn and wheat, and the problem is to maximize the profit.

Expenses	Cost per acre	
	Corn	Wheat
seed	\$12	\$40
fertilizer	\$58	\$80
planting/care/harvesting	\$50	\$90

After the harvest, the farmer must store the crops while awaiting proper market conditions. Each acre yields an average of 110 bushels of corn or 30 bushels of wheat. The limitations of resources are as follows:

Available capital: \$15,000

Available storage facilities: 4,000 bushels

If the net profit (after all expenses have been subtracted) per bushel of corn is \$1.30 and for wheat is \$2.00, how should the farmer plant the 100 acres to maximize the profits?

P = TOTAL PROFIT

$$\begin{aligned}\text{TOTAL PROFIT} &= \text{PROFIT FROM CORN} + \text{PROFIT FROM WHEAT} \\ &= \overbrace{\text{CORN VALUE} \times \text{CORN AMOUNT}} + \overbrace{\text{WHEAT VALUE} \times \text{WHEAT AMOUNT}} \\ &= 1.30 \times 110x + 2.00 \times 30y \\ &= 143x + 60y\end{aligned}$$

$$P = 143x + 60y$$

CONSTRAINTS

$x \geq 0$ The number of acres of corn cannot be negative.

$y \geq 0$ The number of acres of wheat cannot be negative.

These first two assumptions (constraints) will apply in almost every linear programming model.

$x + y \leq 100$ The amount of available land is 100 acres. We do not assume that

$x + y = 100$, because it might be more profitable to leave some land unplanted.

EXPENSES $\leq$ 15,000 The total expenses cannot exceed $15,000.

$$\mathbf{120}x + \mathbf{210}y \leq \mathbf{15{,}000}$$

TOTAL YIELD $\leq$ 4,000 The total yield cannot exceed the storage capacity of 4,000 bushels.

$$\mathbf{110}x + \mathbf{30}y \leq \mathbf{4{,}000}$$

Linear Programming Problem

FARMER PROBLEM

Karl Smith library

Maximize: $P = 143x + 60y$

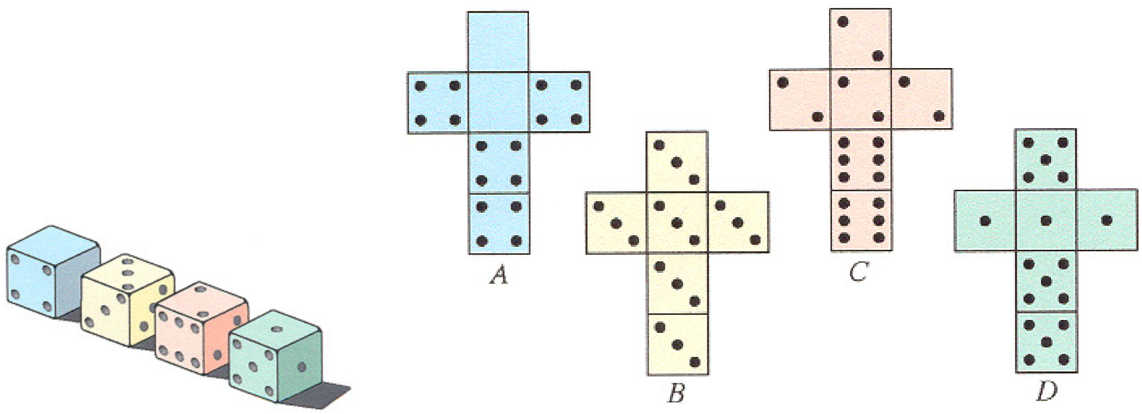
Subject to:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y \leq 100 \\ 120x + 210y \leq 15{,}000 \\ 110x + 30y \leq 4{,}000 \end{cases}$$

VOTING METHODS

Method
Majority method Each voter votes for one candidate. If n is even, then the candidate with $\frac{n}{2}+1$ or more votes wins. If n is odd, then the candidate with $\frac{n+1}{2}$ or more votes wins.
Plurality method Each voter votes for one candidate. Highest vote wins.
Borda count method Each voter ranks the candidates. Last-place is 1 point; next-to-the-last 2 points, and so on. The candidate with the highest number of points wins.
Hare method Each voter votes for one candidate. Majority wins. If no majority, eliminate the least vote candidate.
Pairwise comparison method Each voter ranks the candidates. Each candidate is compared to each of the other candidates; winner of each pairing gets 1 point, and ties get $\frac{1}{2}$ point. The candidate with the most points wins.
Tournament method Compare two at a time, in a predetermined order. Winner of each pairing continues to next round.
Approval method Each voter casts one vote for all the candidates that meet with his or her approval. The candidate with the most votes is declared the winner.

GAME OF WIN

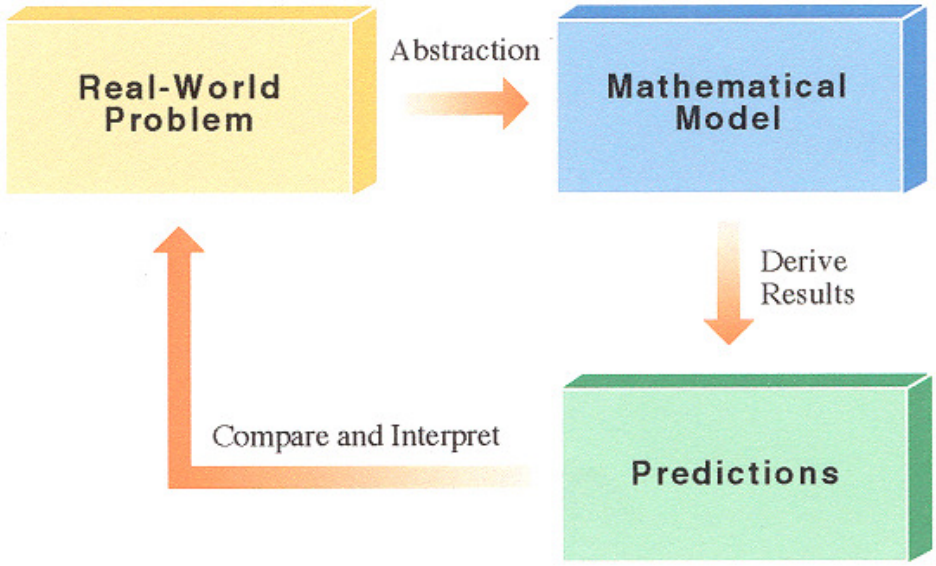


Suppose that one player picks die *A* and that the other picks die *B*. Then we can enumerate the sample space as shown here.

B\A	0	0	4	4	4	4
3	$(3,0)$	$(3,0)$	$(3,4)$	$(3,4)$	$(3,4)$	$(3,4)$
3	$(3,0)$	$(3,0)$	$(3,4)$	$(3,4)$	$(3,4)$	$(3,4)$
3	$(3,0)$	$(3,0)$	$(3,4)$	$(3,4)$	$(3,4)$	$(3,4)$
3	$(3,0)$	$(3,0)$	$(3,4)$	$(3,4)$	$(3,4)$	$(3,4)$
3	$(3,0)$	$(3,0)$	$(3,4)$	$(3,4)$	$(3,4)$	$(3,4)$
3	$(3,0)$	$(3,0)$	$(3,4)$	$(3,4)$	$(3,4)$	$(3,4)$

B wins A wins

MATHEMATICS



math·e·mat·ics \'math-ə-'mat-iks\ n pl but usu sing in constr **1:** the science of numbers and their operations, interrelations, combinations, generalizations, and abstractions and of space configurations and their structure, measurement, transformations, and generalizations **2:** a branch of, operation in, or use of mathematics

This is what it is.