

# TRANSPARENCY MASTERS

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# IT'S ELEMENTARY...

There are four *elementary row operations* for producing equivalent matrices:

1. **RowSwap**

2. **Row +**

3. **\*Row**

4. **\*Row +**



This operation changes only the target row.

## PIVOTING

A process known as pivoting means to carry out the following two steps.

**Step 1** Divide all entries in the row in which the pivot appears (called the pivoting row) by the nonzero pivot element so that the pivot entry becomes a 1. This used elementary row operation **\*Row**.

**Step 2** Obtain zeros above and below the pivot element by using elementary row operation **\*Row +**.

# GAUSS-JORDAN ELIMINATION

- Step 1** Select as the first pivot the element in the first row, first column, and pivot.
- Step 2** The next pivot is the element in the second row, second column; pivot.
- Step 3** Repeat the process until you arrive at the last row, or until the pivot element is a zero. If it is a zero and you can interchange that row with a row below it, so that the pivot element is no longer a zero, do so and continue. If it is zero and you cannot interchange rows so that it is not a zero, continue with the next row.

The final matrix is called the **row-reduced form**.

# Maximizing Profit

A farmer has 100 acres on which to plant two crops, corn and wheat, and the problem is to maximize the profit.

Expenses	Cost per acre	
	Corn	Wheat
seed	\$12	\$40
fertilizer	\$58	\$80
planting/care/harvesting	\$50	\$90

After the harvest, the farmer must store the crops while awaiting proper market conditions. Each acre yields an average of 110 bushels of corn or 30 bushels of wheat. The limitations of resources are as follows:

Available capital: \$15,000

Available storage facilities: 4,000 bushels

If the net profit (after all expenses have been subtracted) per bushel of corn is \$1.30 and for wheat is \$2.00, how should the farmer plant the 100 acres to maximize the profits?

**$P = \text{TOTAL PROFIT}$**

$$\begin{aligned}
 \text{TOTAL PROFIT} &= \text{PROFIT FROM CORN} + \text{PROFIT FROM WHEAT} \\
 &= \overbrace{\text{CORN VALUE} \times \text{CORN AMOUNT}} + \overbrace{\text{WHEAT VALUE} \times \text{WHEAT AMOUNT}} \\
 &= 1.30 \times 110x + 2.00 \times 30y \\
 &= 143x + 60y
 \end{aligned}$$

$$P = 143x + 60y$$

# CONSTRAINTS

$$x \geq 0$$

The number of acres of corn cannot be negative.

$$y \geq 0$$

The number of acres of wheat cannot be negative.



*These first two assumptions (constraints) will apply in almost every linear programming model.*

$$x + y \leq 100$$

The amount of available land is 100 acres. We do not assume that

$x + y = 100$ , because it might be more profitable to leave some land unplanted.

$$\text{EXPENSES} \leq 15,000$$

The total expenses cannot exceed \$15,000.

$$120x + 210y \leq 15,000$$

$$\text{TOTAL YIELD} \leq 4,000$$

The total yield cannot exceed the storage capacity of 4,000 bushels.

$$110x + 30y \leq 4,000$$

# Linear Programming Problem

## FARMER PROBLEM



[www.earlenescakes.com/abtmeimages/FarmerDan.jpg](http://www.earlenescakes.com/abtmeimages/FarmerDan.jpg)

Maximize:  $P = 143x + 60y$

Subject to:

$$\left\{ \begin{array}{l} x \geq 0 \\ y \geq 0 \\ x + y \leq 100 \\ 120x + 210y \leq 15,000 \\ 110x + 30y \leq 4,000 \end{array} \right.$$